Constraining kinematic and dynamic rupture models for broadband strong-motion simulations

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A.-A. Gabriel, J. Premus, J.-P. Ampuero, N. Schliwa, …

Earthquake physics

To model correctly how earthquake ruptures nucleate, propagate, and arrest, we need:

- Laboratory experiments & theoretical considerations
  - Friction laws
  - Simulations & parametric studies -> understanding the roles of individual kinematic and dynamic (stress, friction) parameters

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- Laboratory experiments & theoretical considerations
  - Friction laws
  - Simulations & parametric studies -> understanding the roles of individual kinematic and dynamic (stress, friction) parameters

- Observational constraints
  - Modeling of observed (surface) data -> validation
  - Finding plausible values of kinematic and dynamic parameters (including their spatial heterogeneity) is challenging

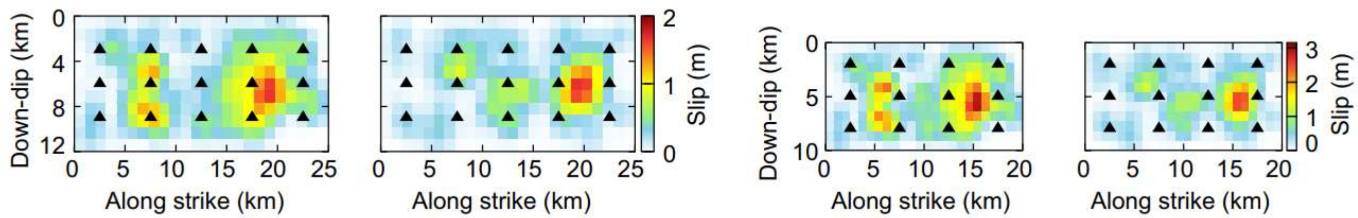
Earthquake finite-fault modeling

Beyond a point-source model:

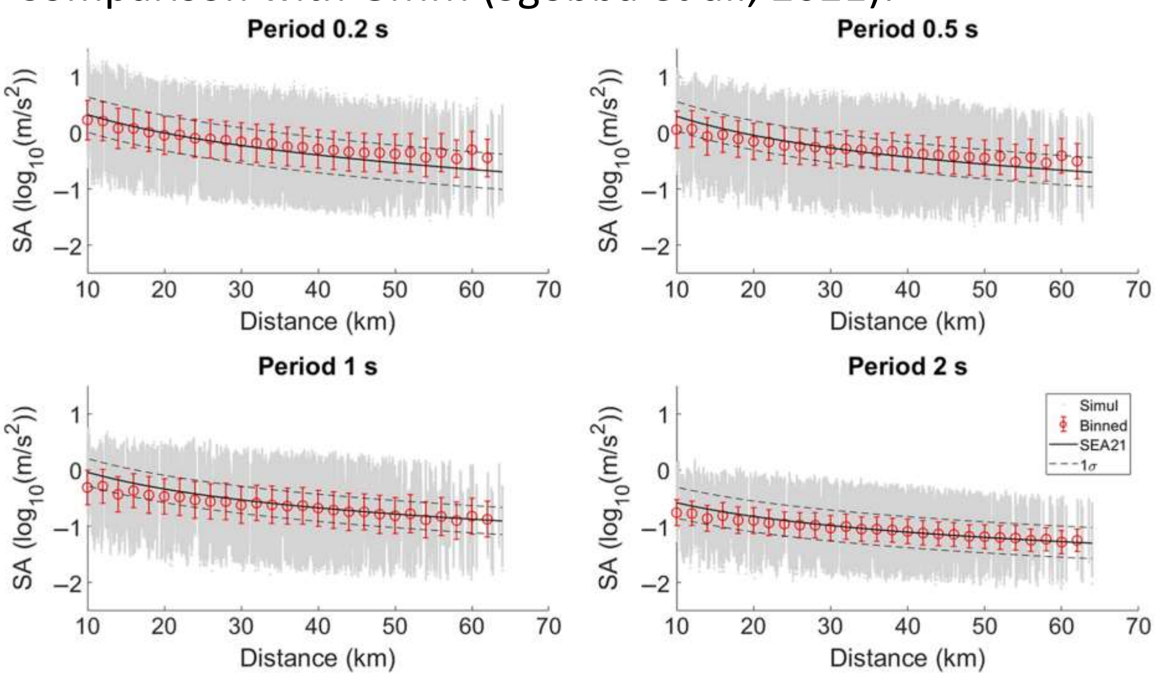
- Kinematic rupture model:
  - Slip rate functions along the fault
  - Linear relation with ground motions (via the representation theorem)
  - Easy to handle, standard in earthquake modeling

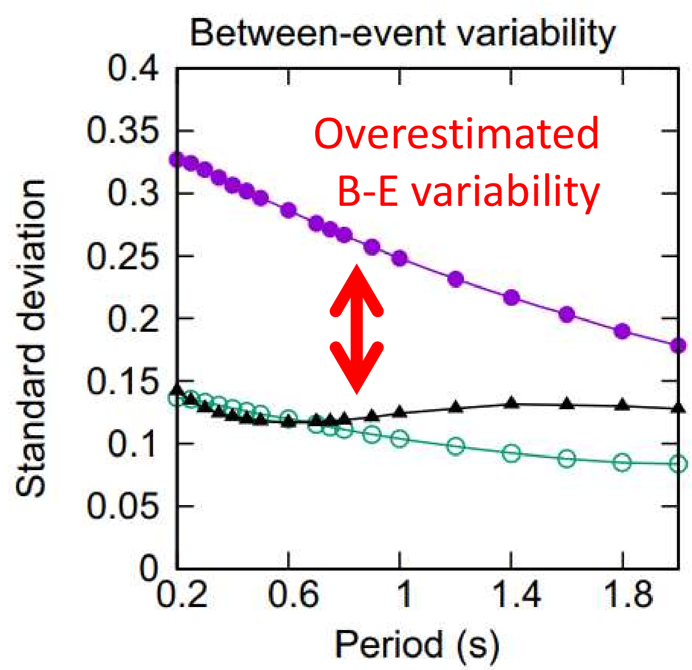
How to constrain scenario variability?

- Čejka et al. (CUNI+INGV Milan, SRL 2024) - Kinematic modeling rupture scenarios of the Amatrice earthquake using HIC
- Variations of slip distribution, size of the rupture, nucleation point, rupture speed



Comparison with GMM (Sgobba et al., 2021):

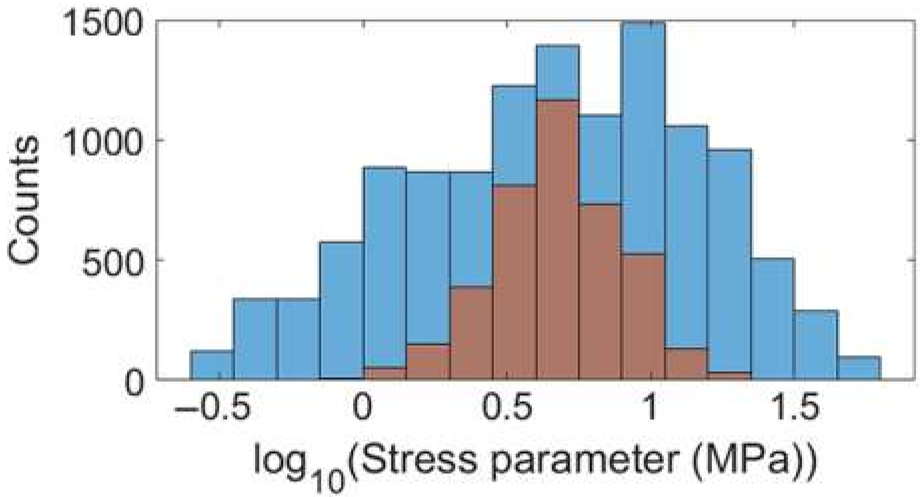


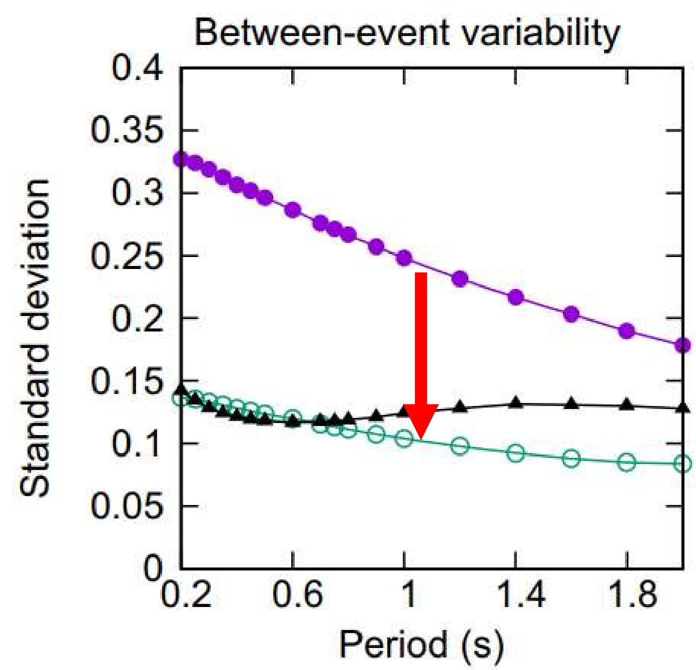


How to constrain scenario variability?

- Čejka et al. (CUNI+INGV Milan, SRL 2024) - Kinematic modeling of rupture scenarios of the Amatrice earthquake using HIC
- Using the rejection method, we randomly pick scenarios and accept or reject them stochastically based on their B-E residual
- From the scaling of the acceleration plateau, we can obtain an equation for the stress parameter:

$$\Delta\sigma = \frac{7}{16}\left(\frac{a}{k}\frac{v_r}{v_s}\right)^3 \frac{M_0}{S^{3/2}}$$





Earthquake finite-fault modeling

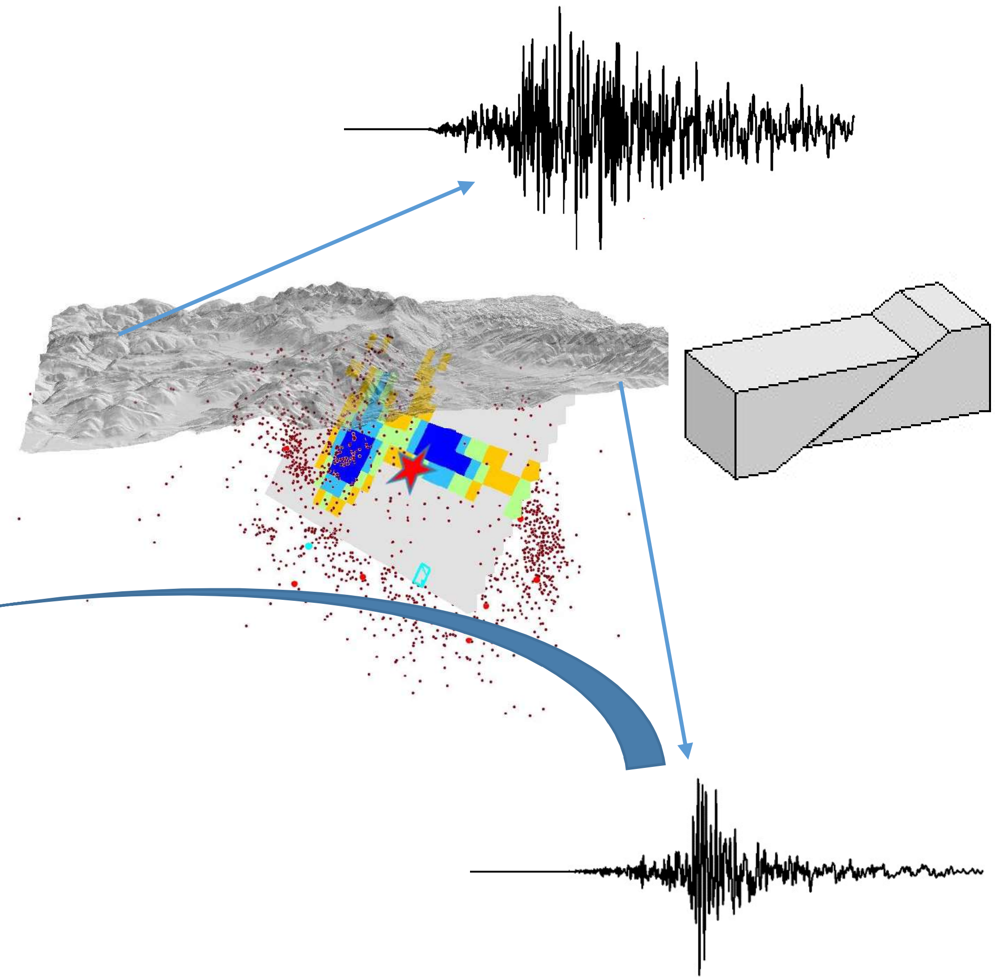
Beyond a point-source model:

- Kinematic rupture model:
  - Slip rate functions along the fault
  - Linear relation with ground motions (via the representation theorem)
  - Easy to handle, standard in earthquake modeling

- Dynamic rupture model (physics-based):
  - Friction law + elastodynamics
  - Non-linear relation with ground motions (via rupture simulation and representation theorem)
  - Cumbersome in some applications, still waiting for widespread use

Dynamic slip inversion

Elastodynamic equation and *a friction law* act as physical constraints

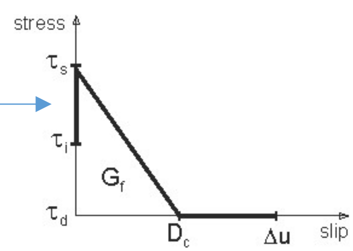


Fault plane

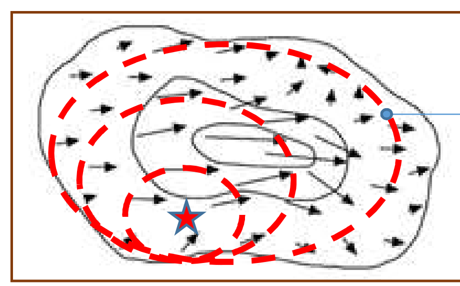
Distribution of

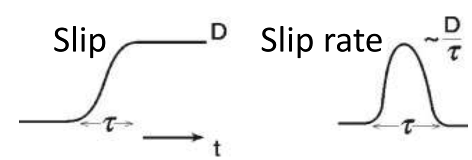
- prestress
- friction parameters

Friction law



Dynamic rupture simulation



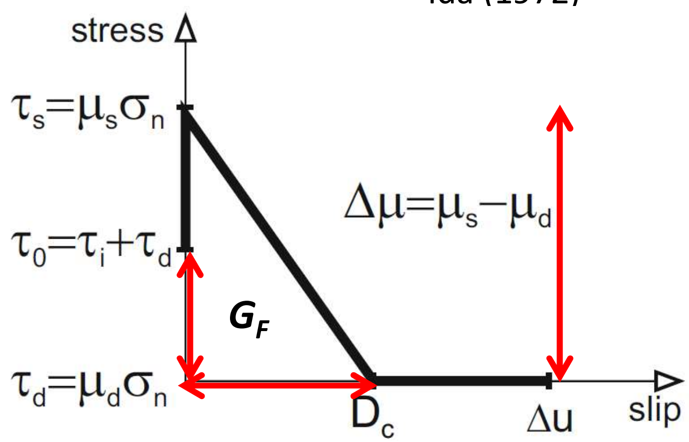


Friction law – linear slip weakening

Fault plane

Distribution of
- prestress
- friction parameters

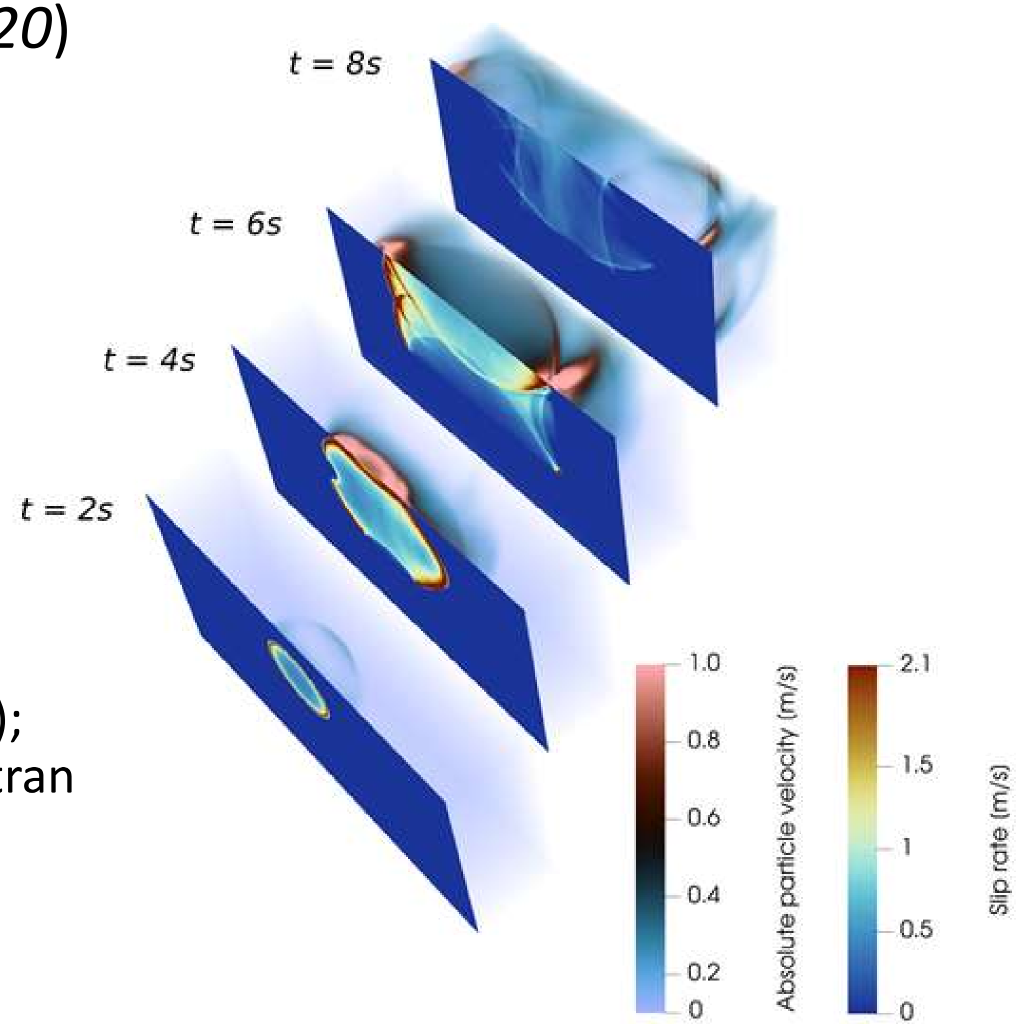
Ida (1972)



- Model parameters (inhomogeneous along the fault):
  - **Friction drop $\Delta\mu$** (static-dynamic friction coefficient)
  - **Characteristic slip-weakening distance D_c**
  - **Initial stress τ_i**

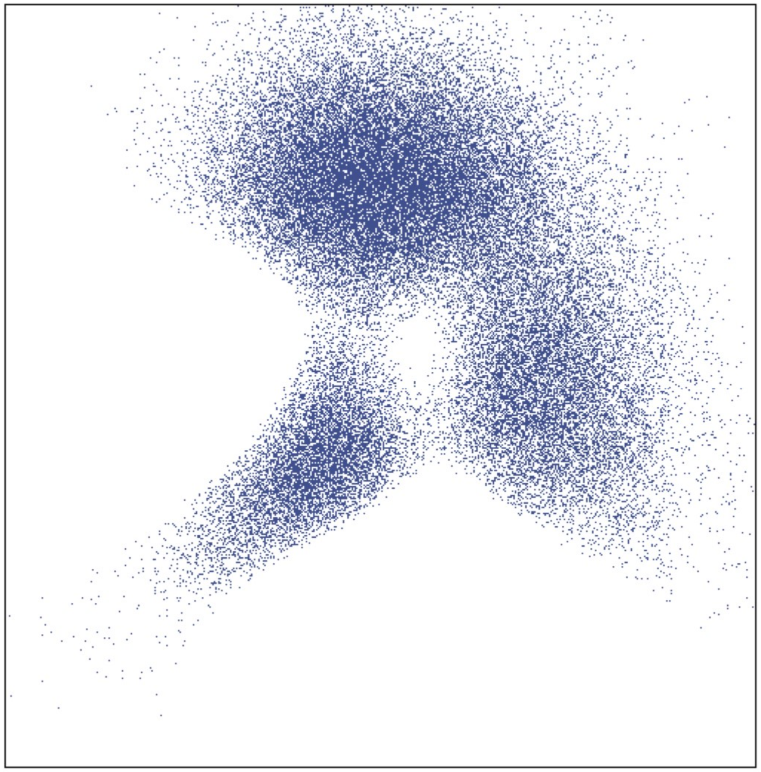
Forward solver (speed matters)

- FD3D_TSN (*Premus et al., 2020*)
- Community test with heterogeneous dynamic parameters
- Fault size 30x15km (grid step 100m)
- 12s of rupture propagation calculated in:
  - 3min on 1 CPU (Intel i9-9900K)
  - 10s on 1 GPU (Nvidia GTX 4700); ported using OpenACC in nvfortran
- Freely available on GitHub



Bayesian inversion by Monte Carlo approach

- **Data errors**: Gaussian distribution with constant variance
- **Prior information** (constraints) on model parameters:
  - Dc > … m (for numerical reasons)
  - Nonnegative friction drop and prestress
  - Nucleation (negative strength excess) only around the prescribed hypocenter
- **Posterior PDF** is sampled using Monte Carlo Markov Chain by the Parallel tempering method (Sambridge, 2013)



Applications of Bayesian dynamic rupture inversions

- 2019 Mw6.2 Amatrice (Central Italy)
  - Gallovič et al. (JGR 2019b)
- 2020 Mw 6.8 Elazığ (Turkey)
  - Gallovič et al. (CommEE 2020)
- 2014 Mw 6.0 South Napa (California)
  - Premus et al. (Science Advances, 2022)
- 2017 Mw 6.3 Lesvos (Greece)
  - Kostka et al. (GJI 2022)
- 2011 and 2016 Mw 5.8 Ibaraki twins (Japan)
  - Gallovič (in preparation)
- 2004 Mw 6 Parkfield (California)
  - Schliwa et al. (JGR 2024)
- 2016 Mw 6.2 Tottori (Japan)
  - M. Hronek (in preparation)

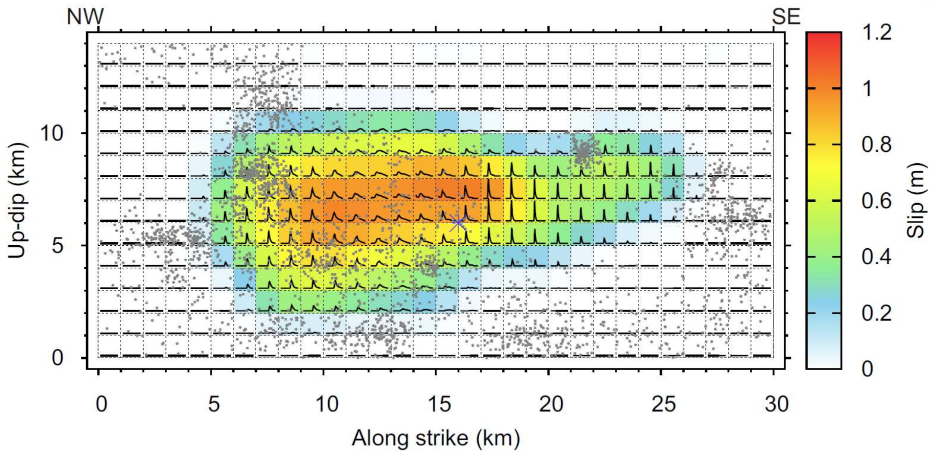
Dynamic rupture inversions of large crustal earthquakes

Published showcase examples:

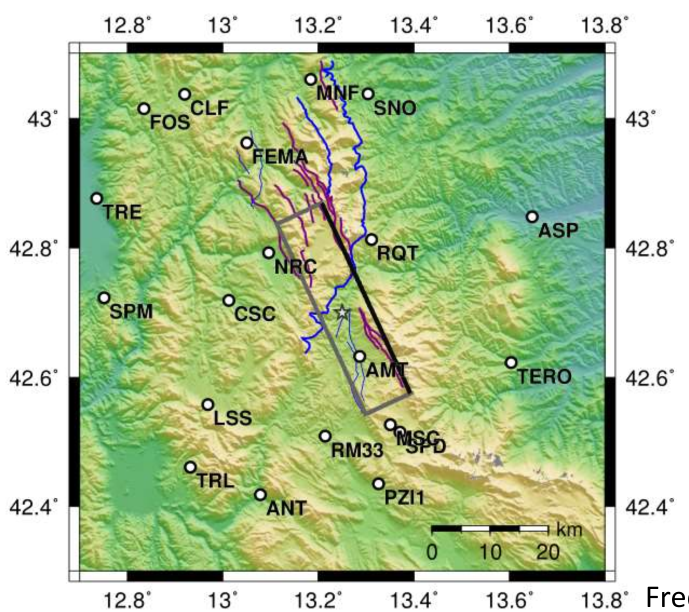
2016 Mw6.2 Amatrice, 2004 Mw6 Parkfield, 2014 Mw6 South Napa

Best-fitting model from the Monte Carlo inversion (out of ~10^6 visited models)

Gallovič et al. (2019, JGR)

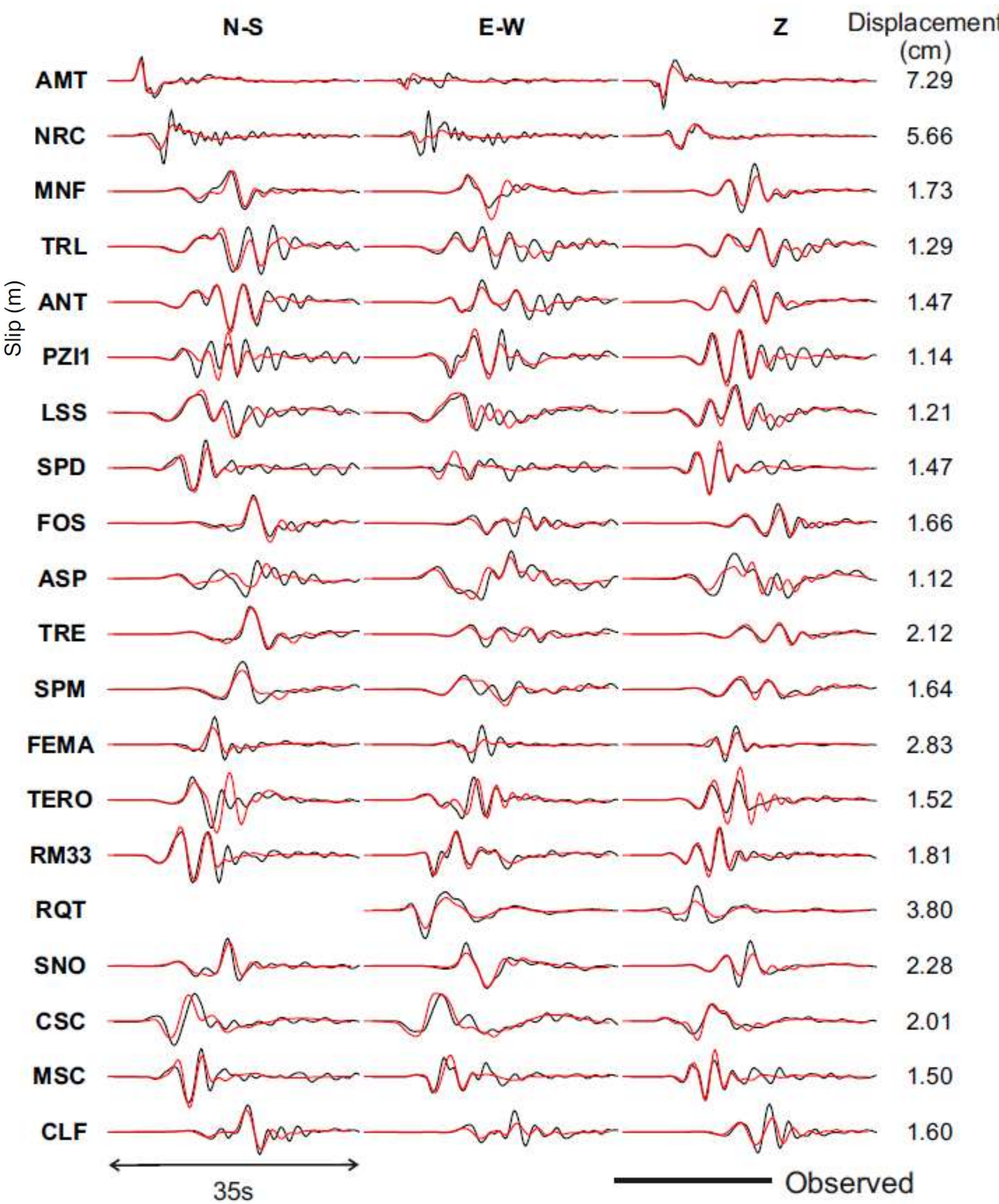


On-fault aftershocks relocated by Chiaraluce et al. (2017)

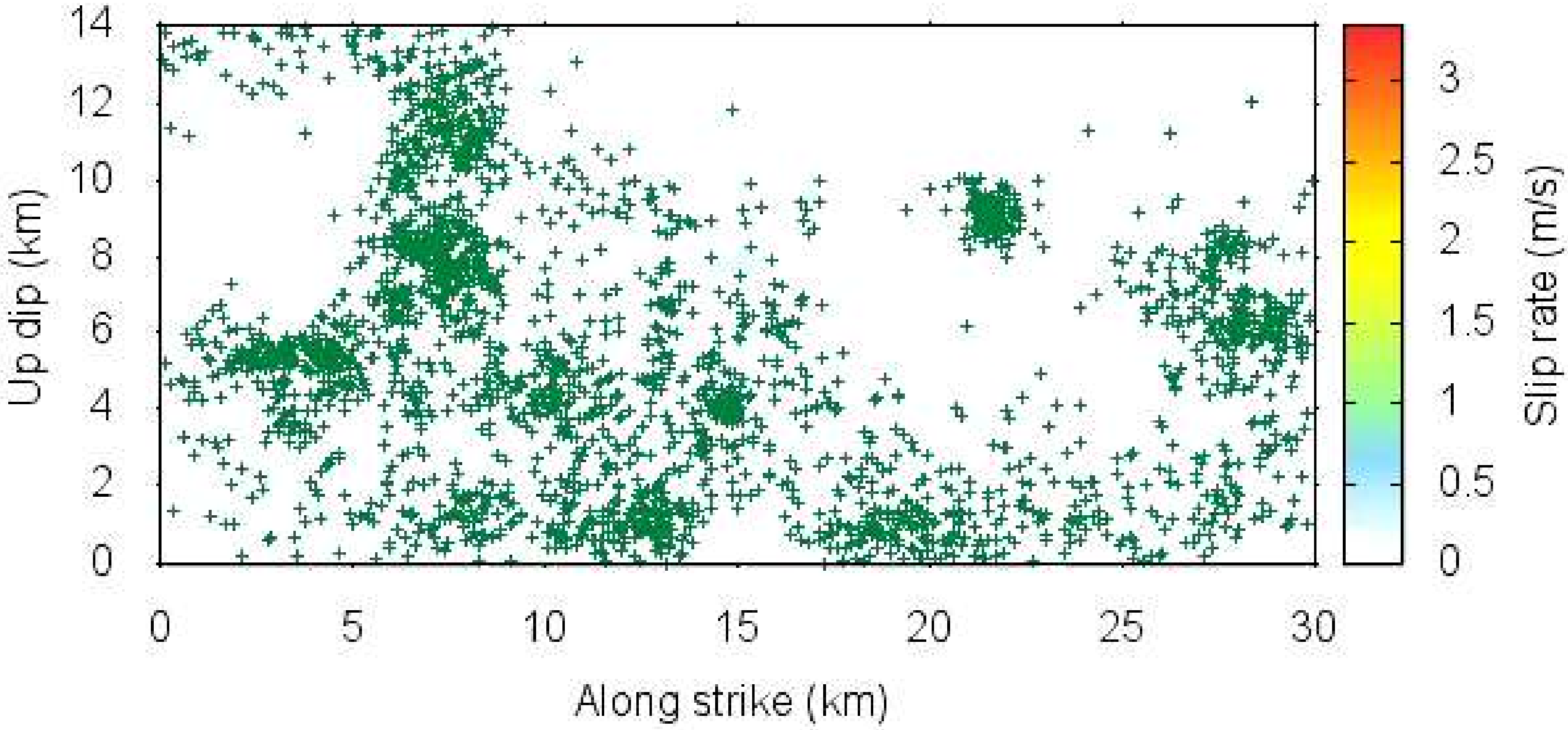


VR 62%

Forward simulation: 2 mins on CPU, 30s on GPU

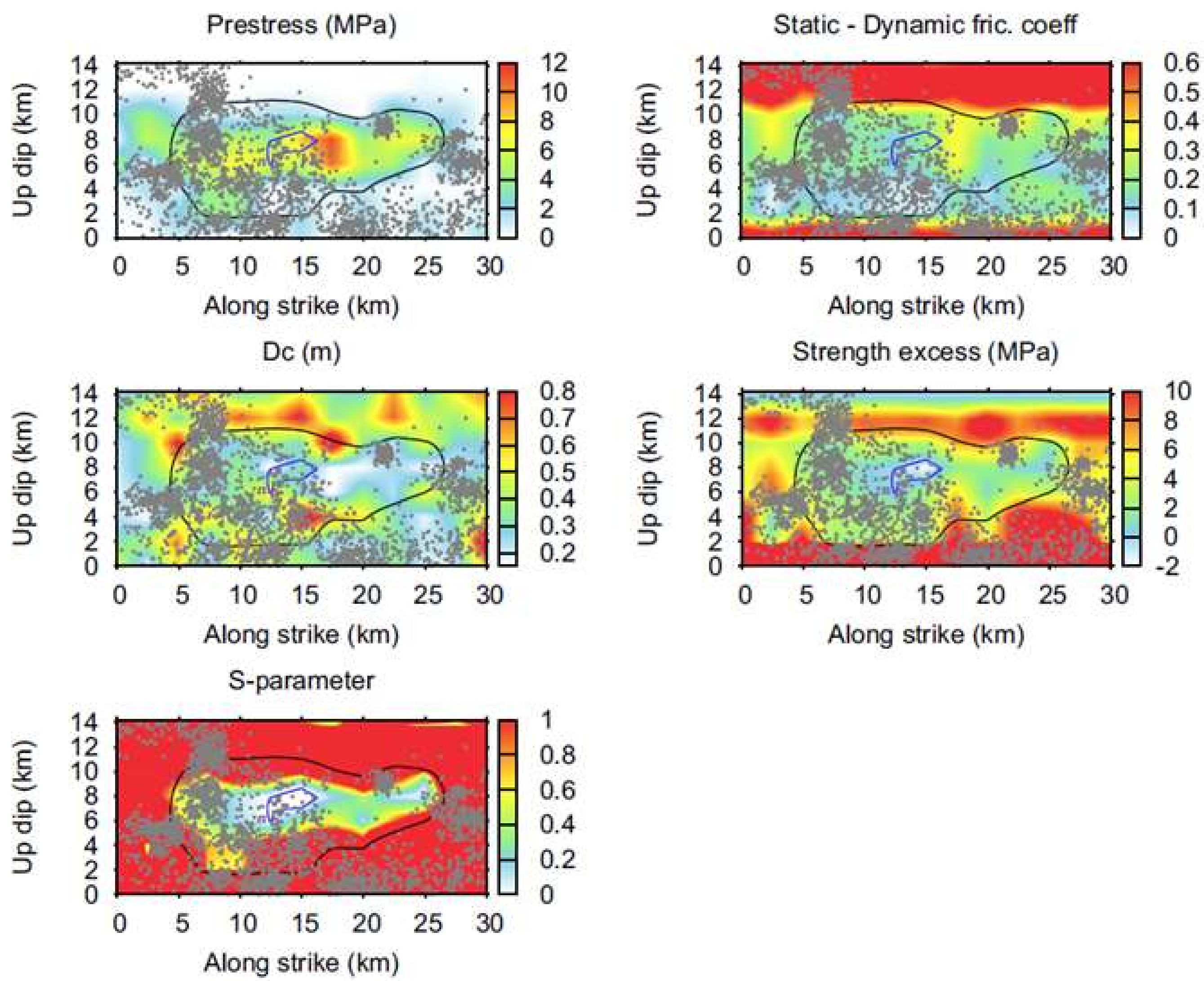


Frequency range from 0.05 to 1Hz (AMT, NRC) and 0.5Hz (others)



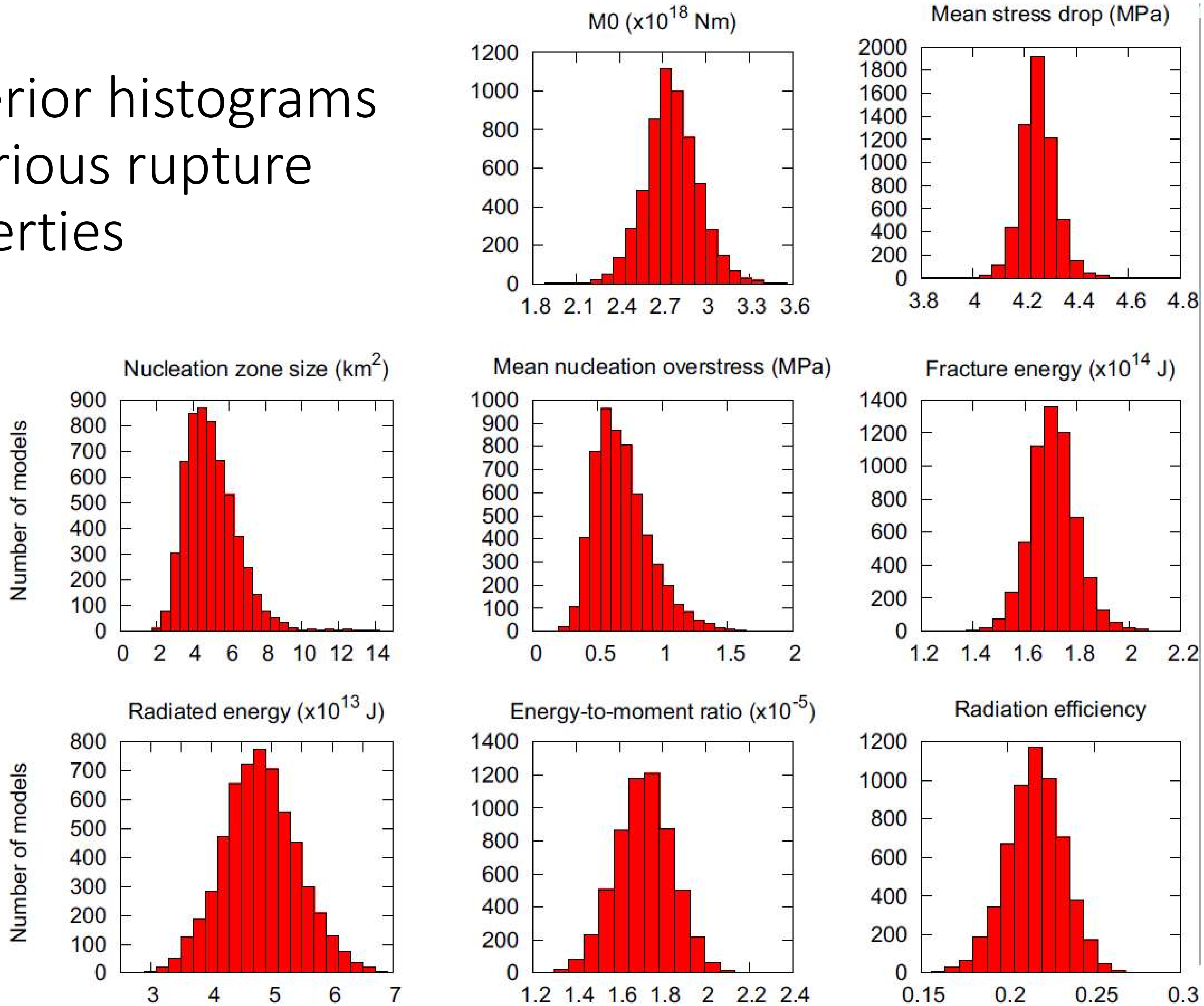
14
12
10
8
6
4
2
0
Up dip (km)
0
5
10
15
20
25
30
Along strike (km)
3
2.5
2
1.5
1
0.5
0
Slip rate (m/s)

Dynamic parameters of our best-fitting model



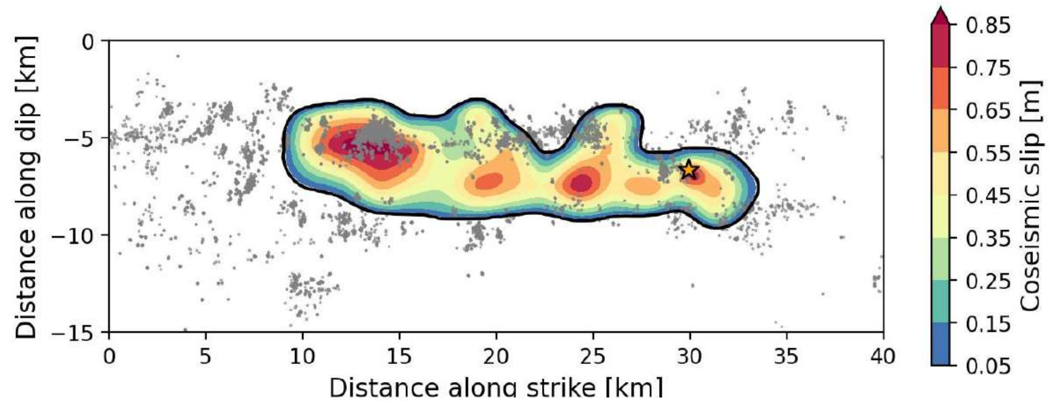
Black contour – slip contour
Blue contour – nucleations patch

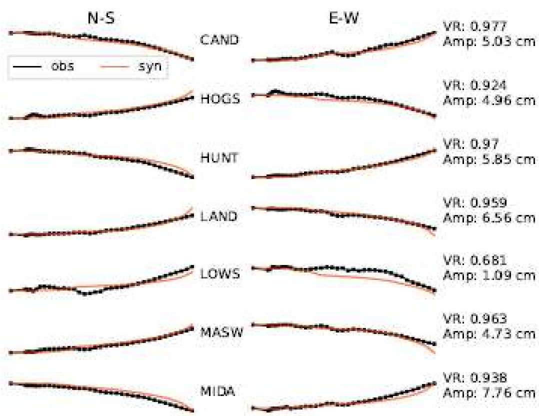
Posterior histograms of various rupture properties

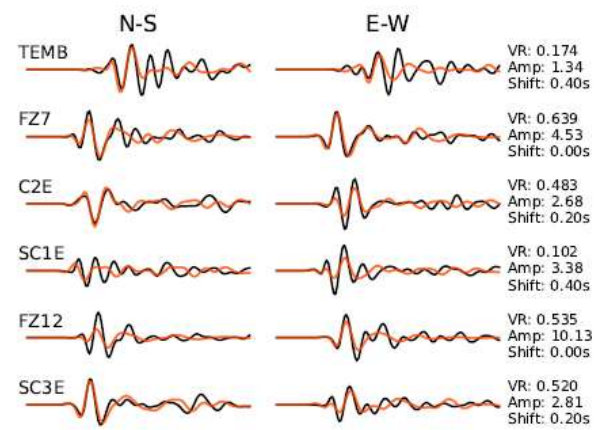


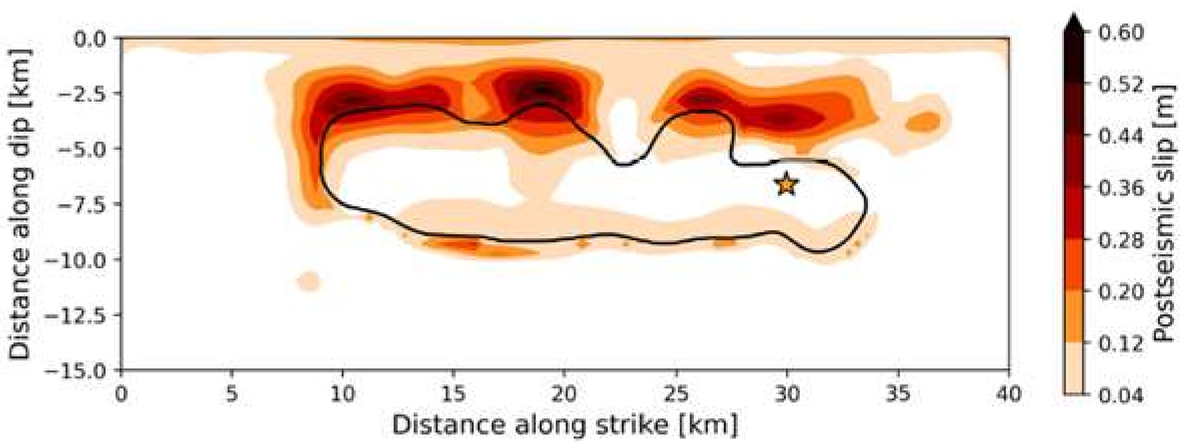
Joint inversion of co- and post-seismic slip

- 2004 Mw 6 Parkfield (California)
- Schliwa et al. (JGR, 2024)
- Friction law: rate&state with rapid velocity weakening



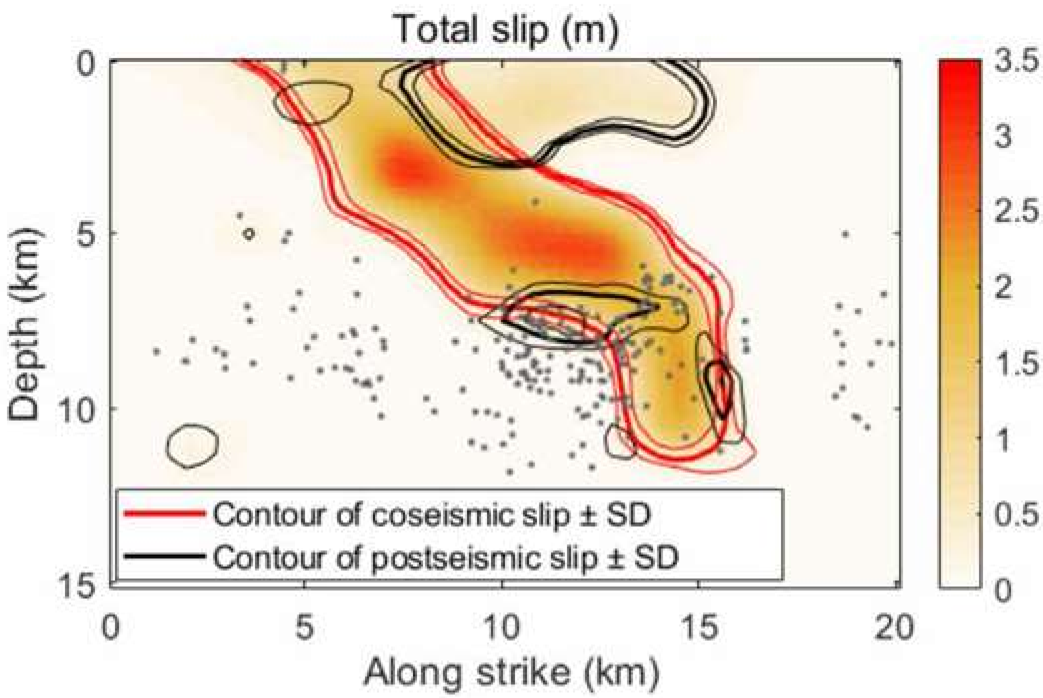


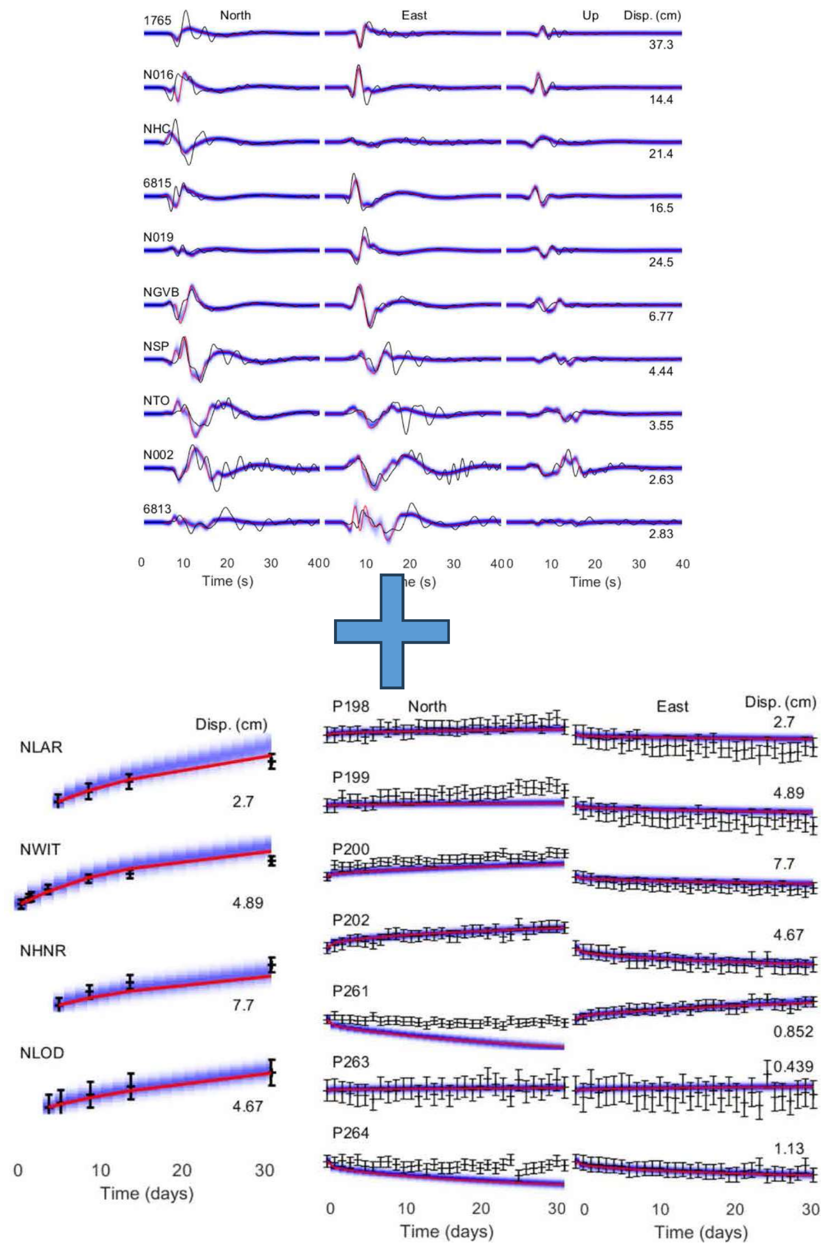




Joint inversion of co- and post-seismic slip

- 2014 Mw 6.0 South Napa (California)
- Premus et al. (Science Advances, 2022)
- Friction law: rate&state with rapid velocity weakening

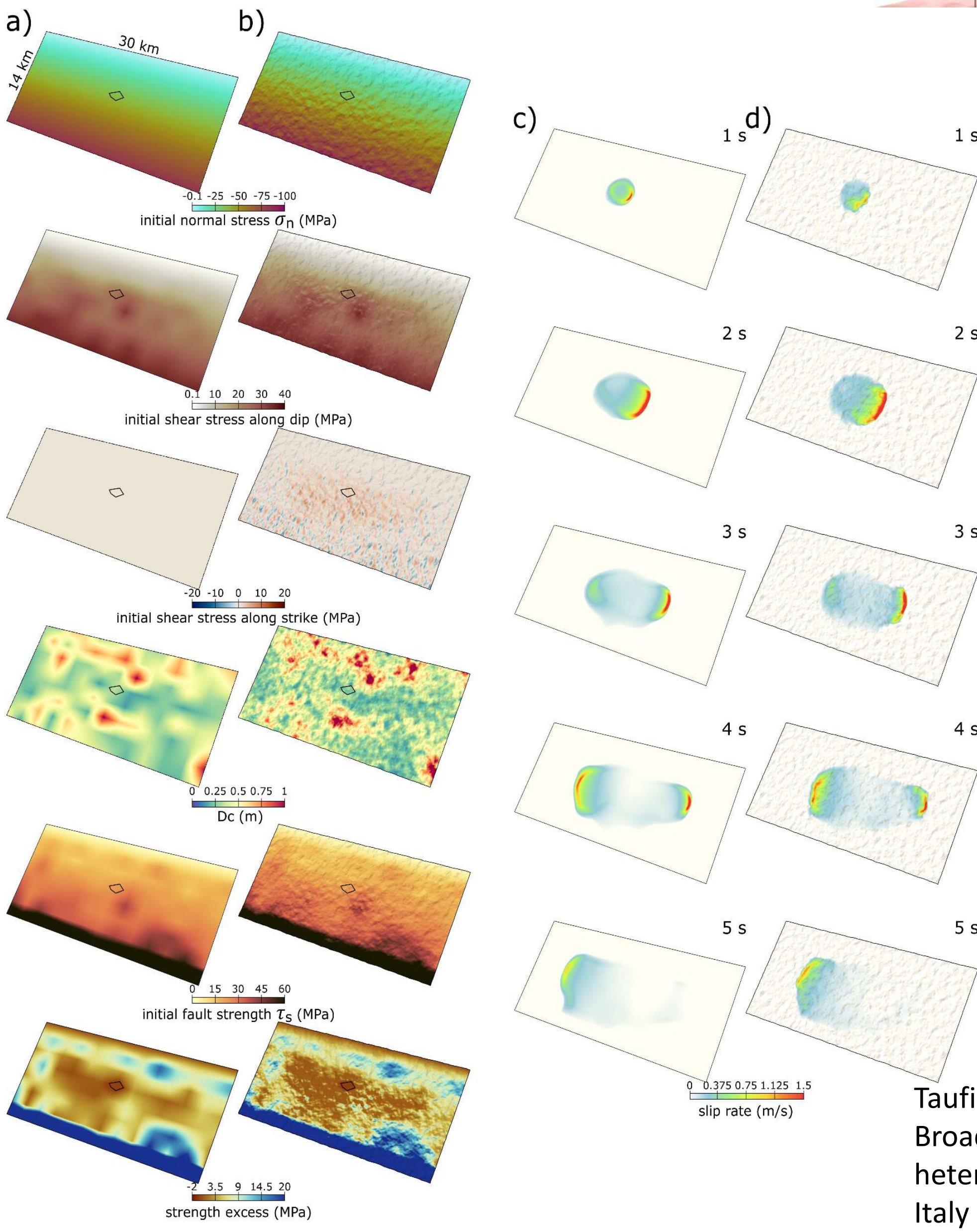


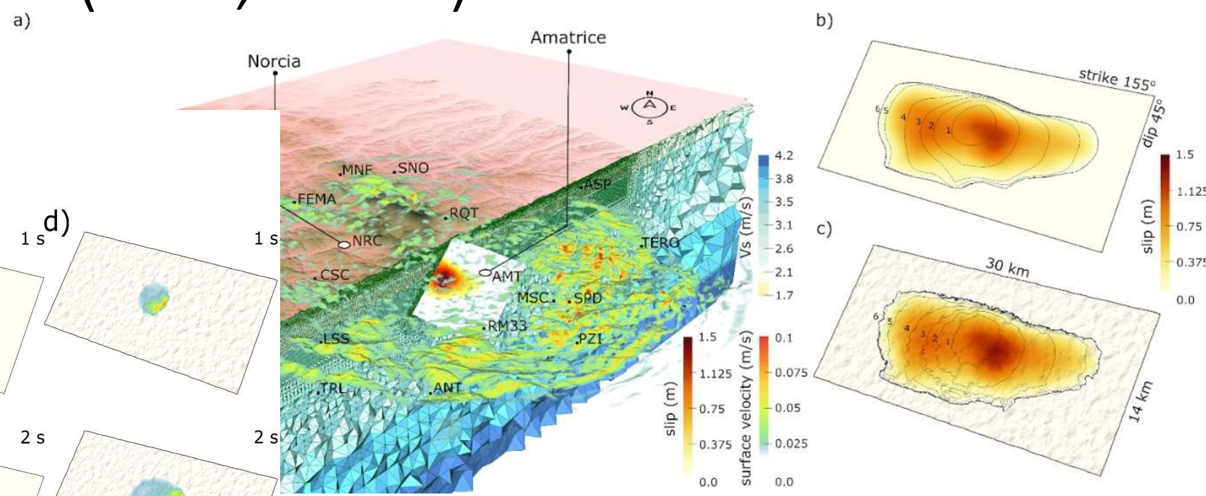


Towards broadband ground motion simulations

(how to introduce small-scale heterogeneity to dynamic rupture models)

Taufiqurrahman et al. (GRL, 2022)





“ More than 80 million elements. ”

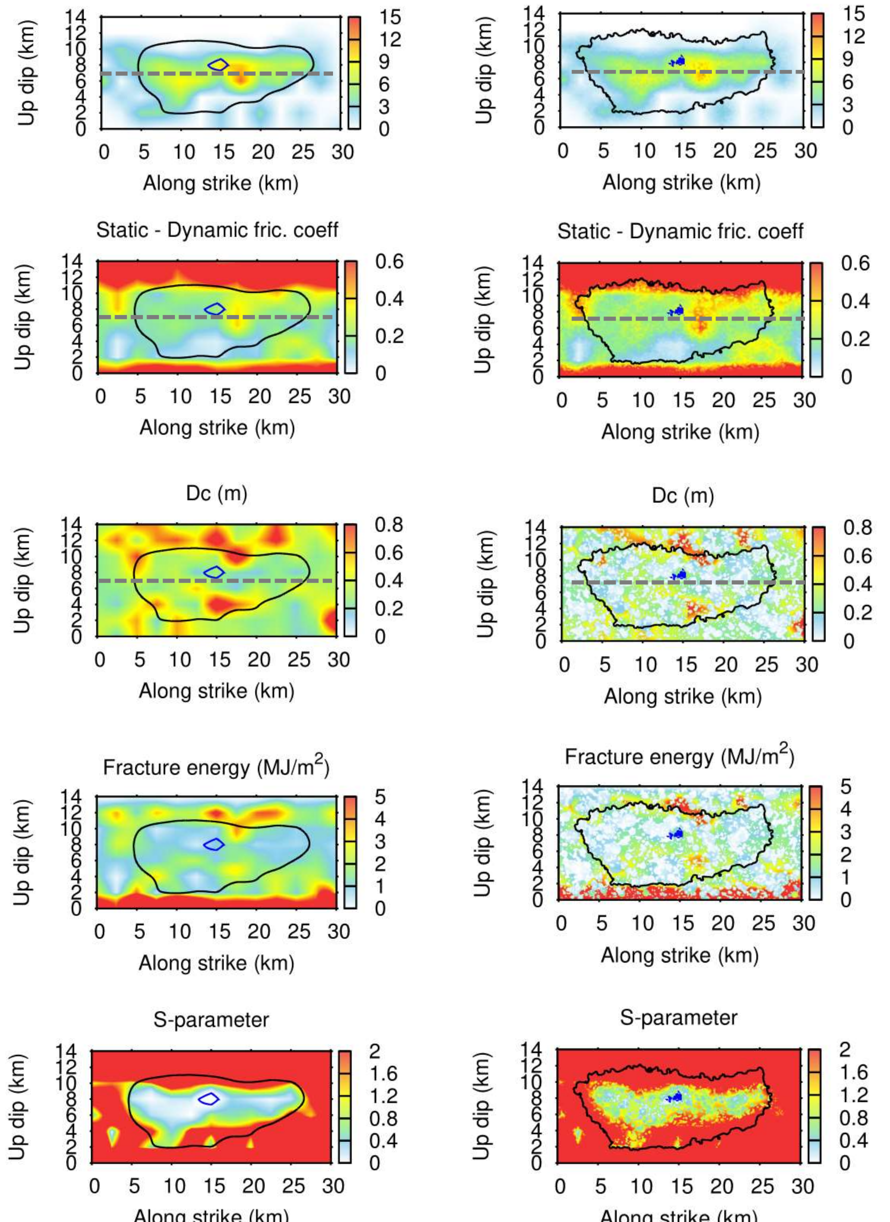
“Simulating 40 s of a broadband Amatrice dynamic rupture earthquake scenario using SeisSol requires 4 hours on 256 **nodes** of the SuperMUC-NG supercomputer.”

Taufiqurrahman, A.-A. Gabriel, T. Ulrich, Ľ. Valentová, F. Gallovič (2022). Broadband dynamic rupture modeling with fractal fault roughness, frictional heterogeneity, viscoelasticity and topography: the 2016 Mw 6.2 Amatrice, Italy earthquake, Geophys. Res. Lett. 49, e2022GL098872.

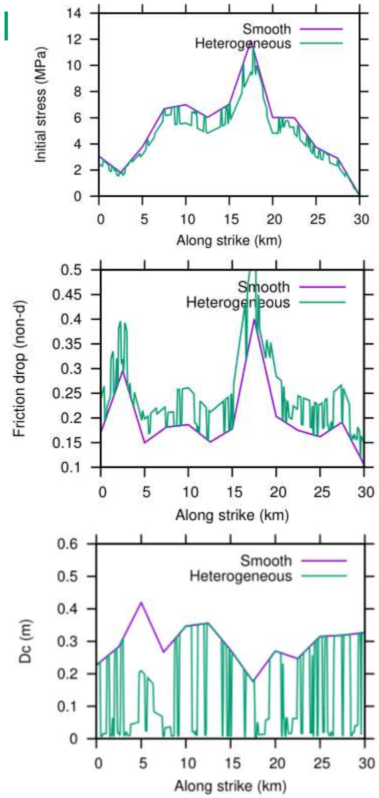
Example – Amatrice (Gallovič and Valentová, JGR 2023)

Smooth

Fractal



Gallovič et al. (JGR 2019)

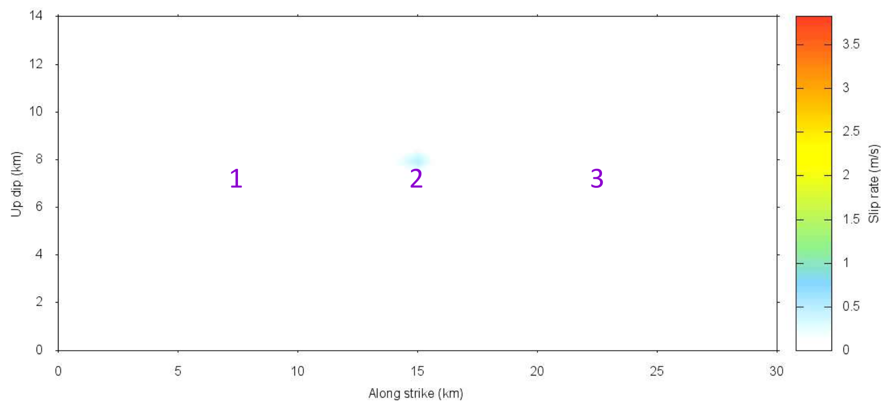


Added heterogeneities follow the Ide and Aochi (2005) fractal Dc model

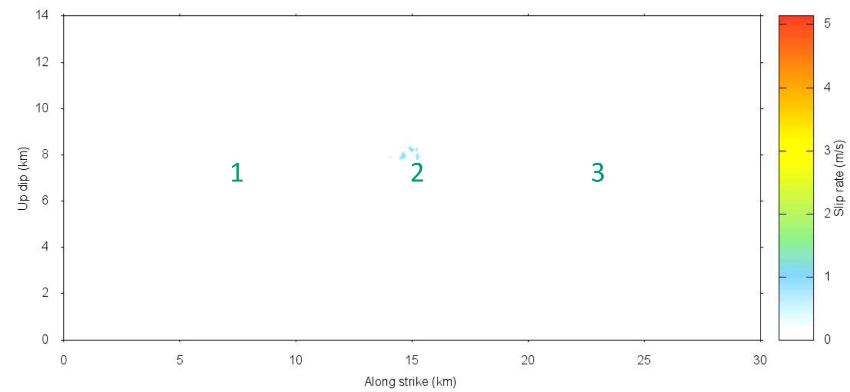
Note – the specific realization of the fractal distribution found to obtain the best fit with low-frequency seismograms (<0.5Hz) out of 500 random realizations.

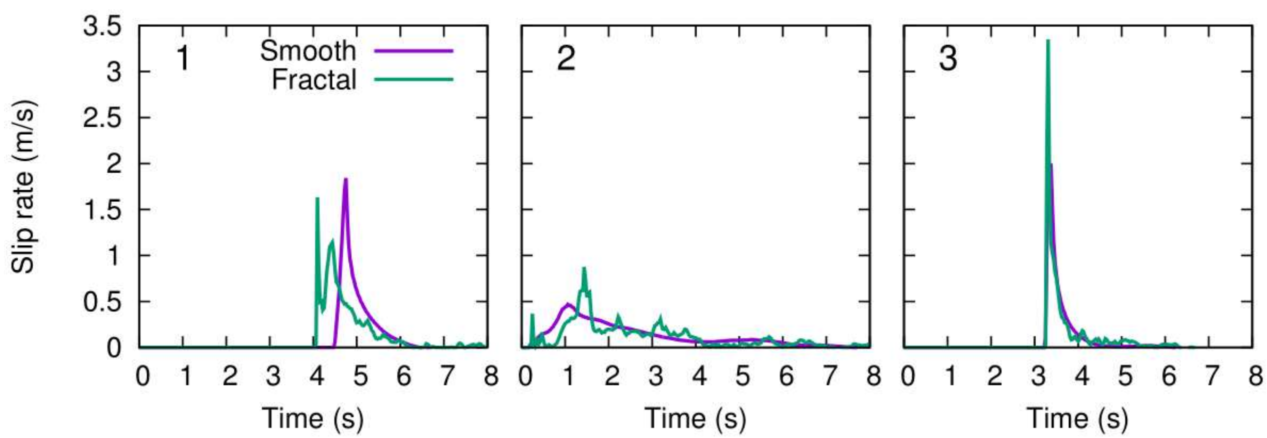
Example – Amatrice

Smooth



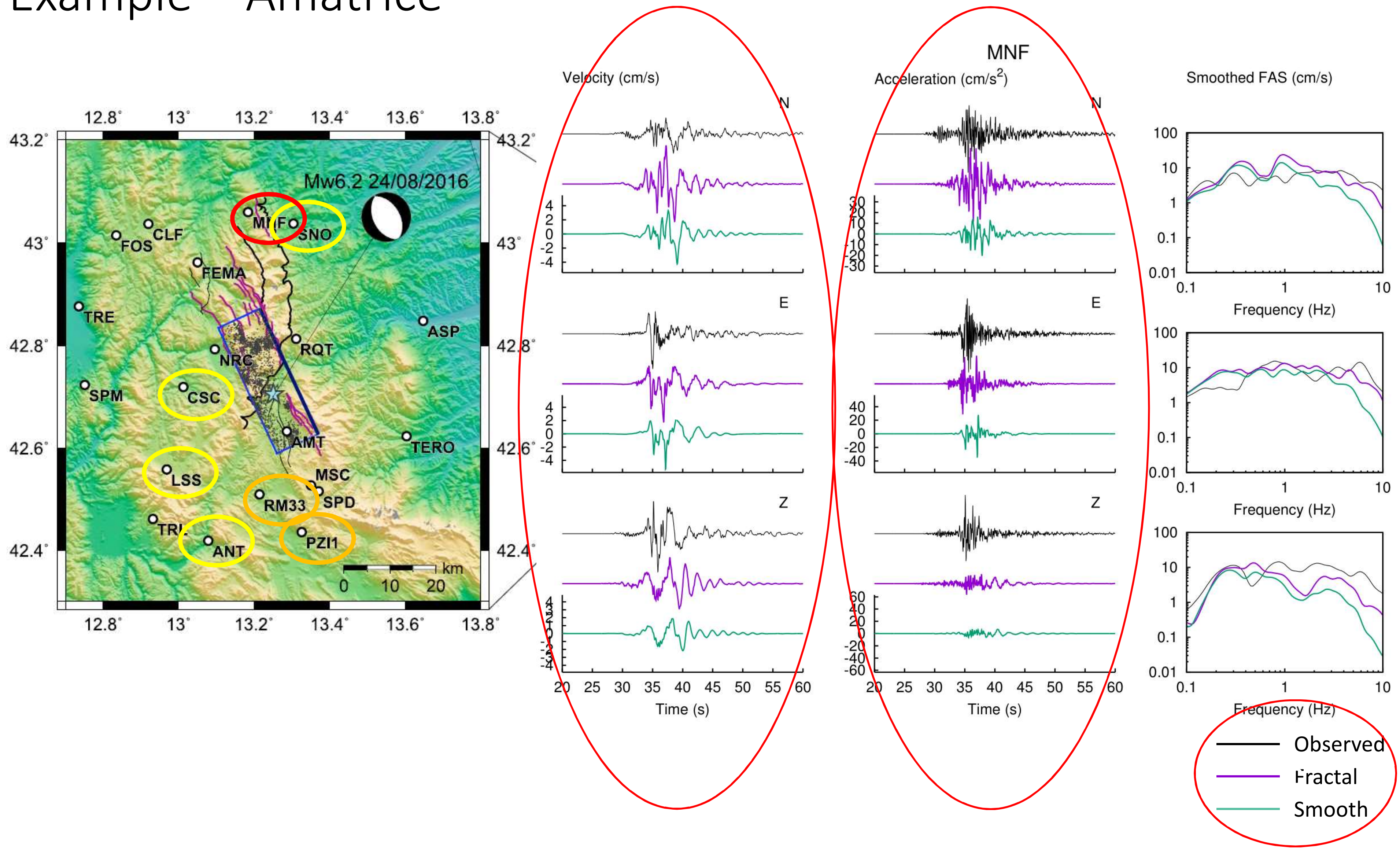
Fractal



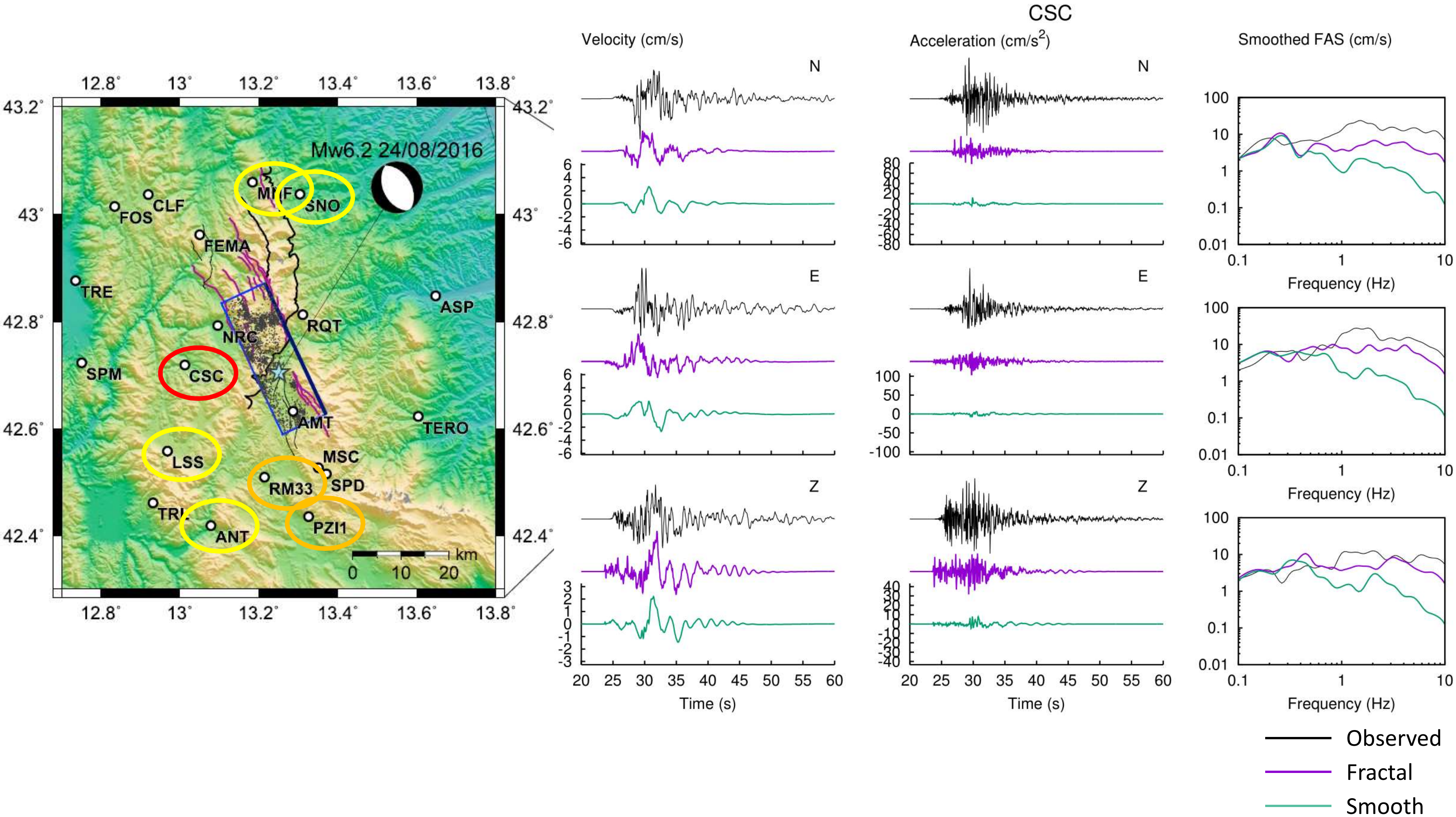


FD3D_TSN performs the calculation in about
30 minutes up to 10 Hz on a single GPU

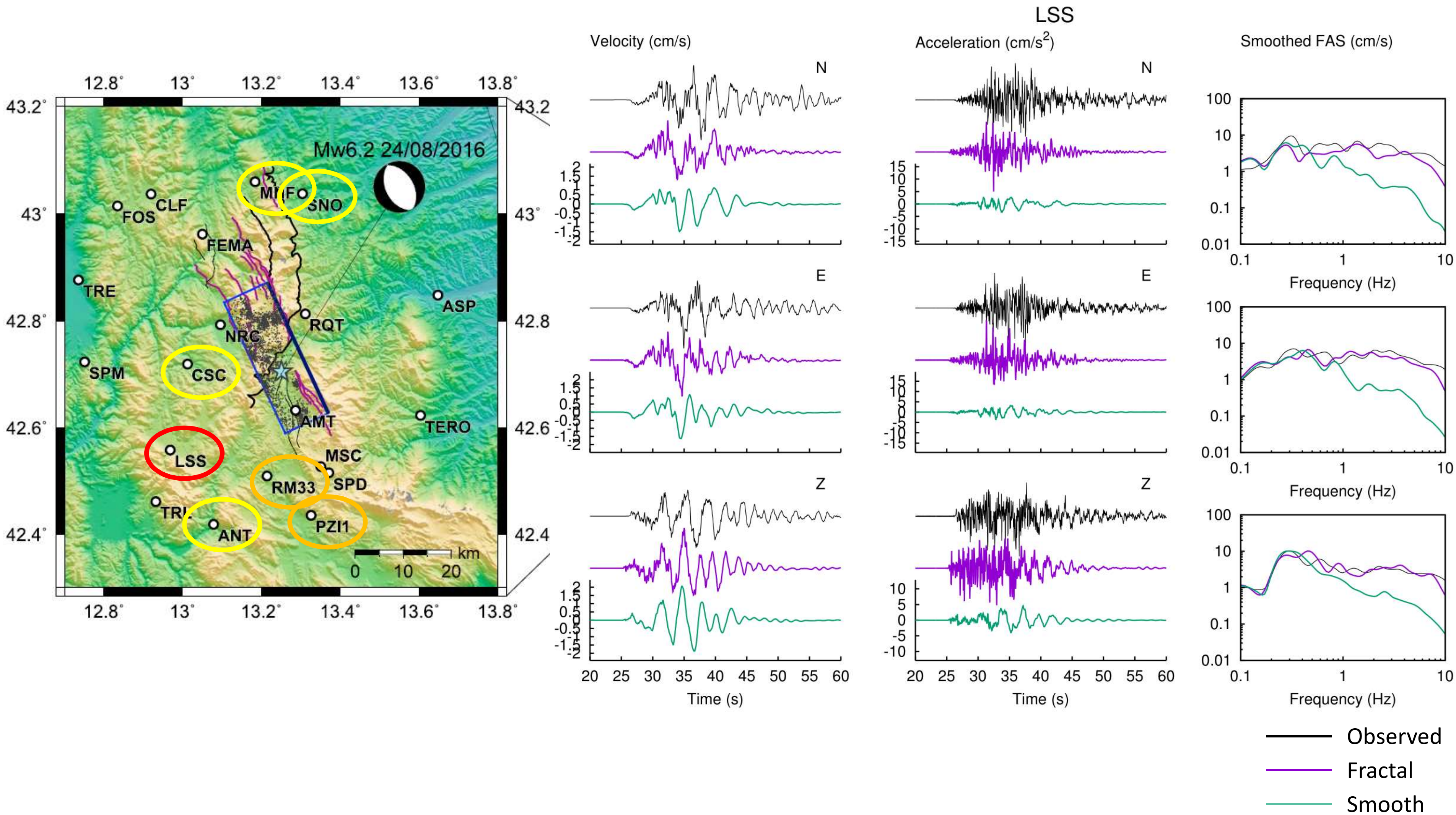
Example – Amatrice



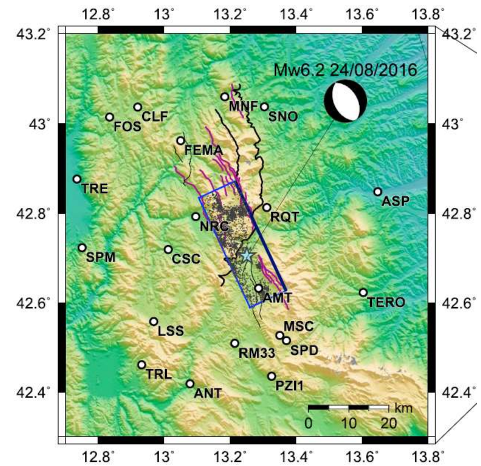
Example – Amatrice



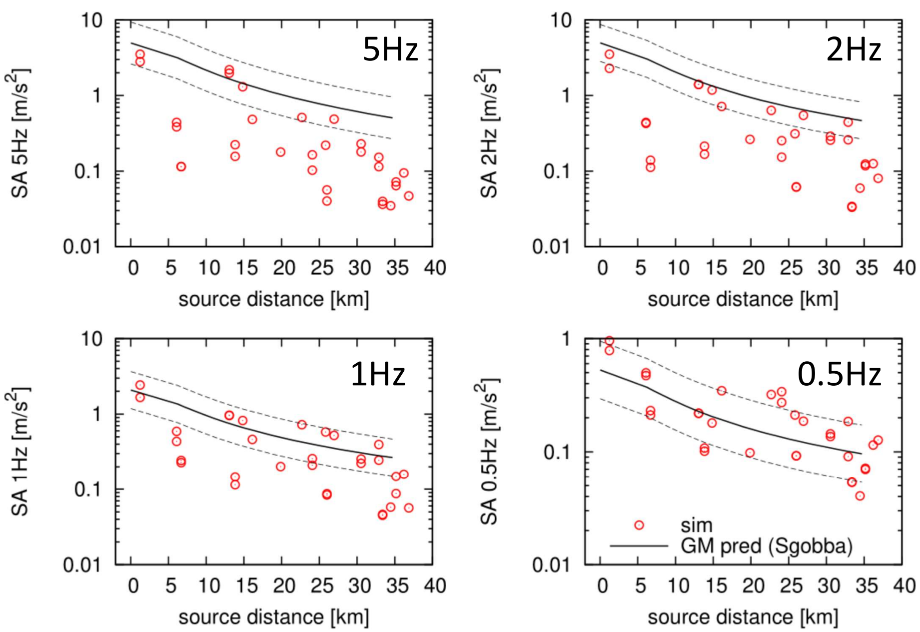
Example – Amatrice



Example – Amatrice (comparison with Ground Motion Model)

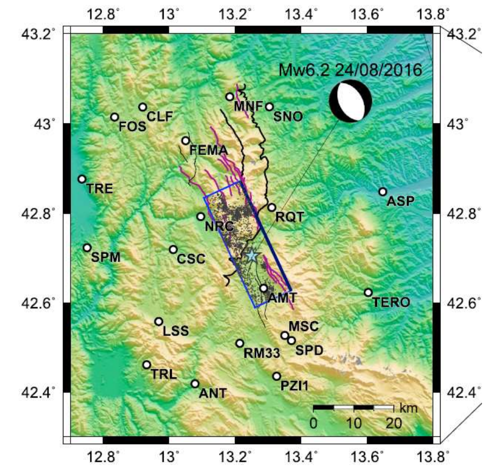


Smooth



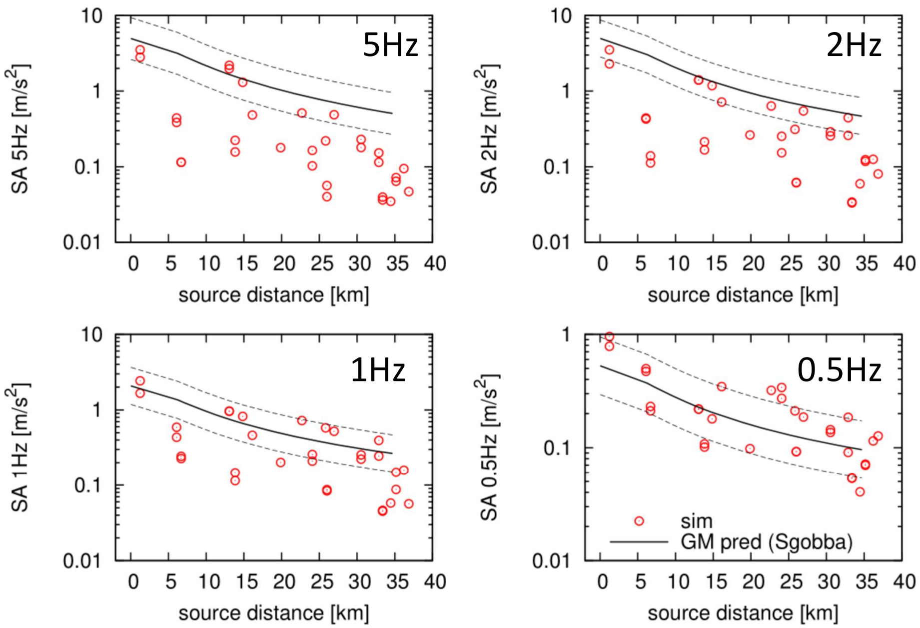
GM pred: Sgobba et al. (2021)

Example – Amatrice (comparison with Ground Motion Model)

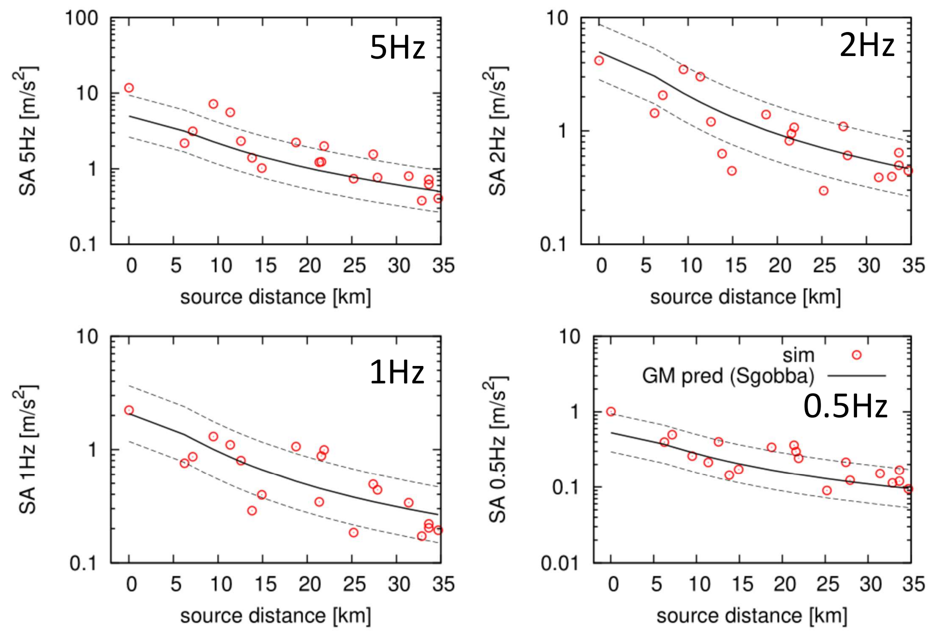


Gallovič, F., Valentová, Ľ. (2023). Broadband strong ground motion modeling using planar dynamic rupture with fractal parameters, J. Geophys. Res. Solid Earth 128, e2023JB026506.

Smooth



Fractal



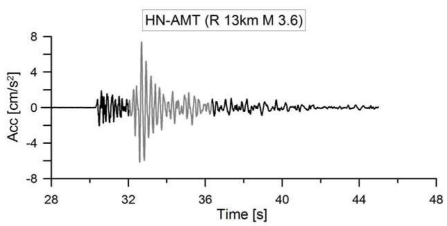
GM pred: Sgobba et al. (2021)

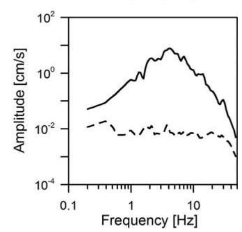
Beyond the Brune model: Dynamic rupture inversion of apparent source spectra of small earthquakes

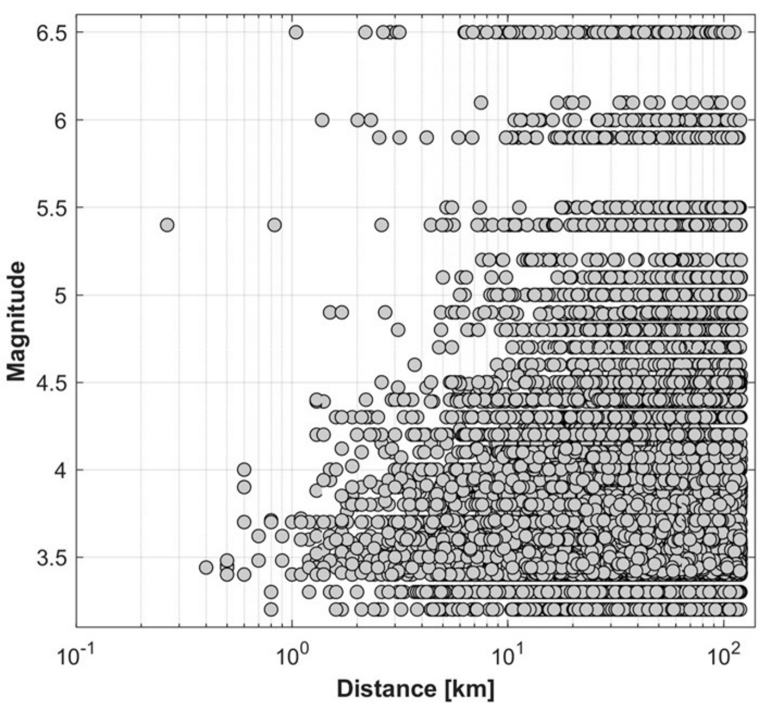
Gallovič, F., Sgobba, S., Valentová K., Ľ.: Physics-based broadband characterization of weak earthquakes, Nature Communications (2025)

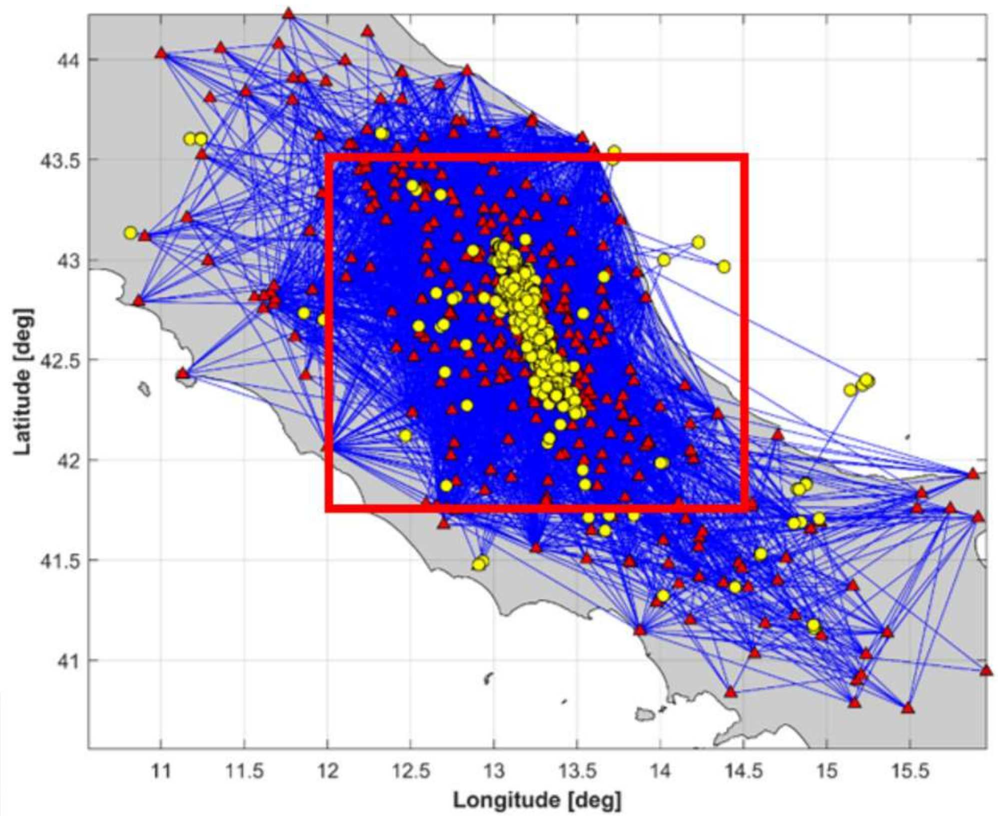
Data

GIT method (Castro et al. 1990; Bindi et al., 2009; Pacor et al., 2016; Oth et al., 2008)





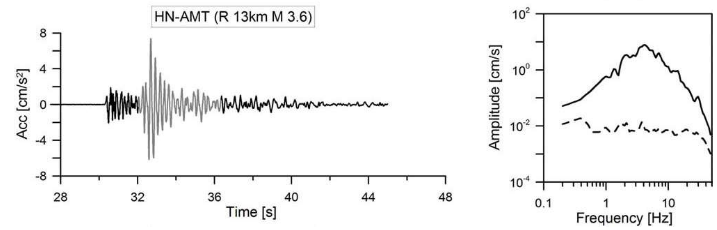




We exploit non-parametric GIT decomposition performed over many crustal events and stations in Central Italy, considering amplitude spectra at 56 frequencies equally spaced on a logarithmic scale in the **0.5-25 Hz** range by

Morasca et al. (2023), Colavitti et al. (2022), Spallarossa et al (2021)

Data



$$\log A_{ij}(f, r_{ij}) = \log \tilde{S}_i(f) + \log P_{ij}(f, r_{ij}) + \log G_j(f) + \delta W_{ij}(f)$$

- A_{ij} is the geometric mean of horizontal components of the Fourier amplitude spectra of dominant S-wave packets in acceleration of event *i* and station *j*, smoothed by the Konno and Ohmachi (1998) algorithm
- $\tilde{S}_i$ corresponds to the source term
- P_{ij} is the attenuation function comprising the geometrical spreading and frequency-dependent attenuation
- G_j is the site response
- δW_{ij} is the residual term from the GIT spectral fitting

- P_{ij} is considered equal to 1 at distance $r_{ij} = r_{ref} = 10$ km
- One or more stations are carefully selected as reference rock stations for which no site response is assumed ($G_j = 1$).

Data

$$\log A_{ij}(f, r_{ij}) = \log \tilde{S}_i(f) + \log P_{ij}(f, r_{ij}) + \log G_j(f) + \delta W_{ij}(f)$$

$$\log S_{ij} = \log \tilde{S}_i(f) + \delta W_{ij}$$

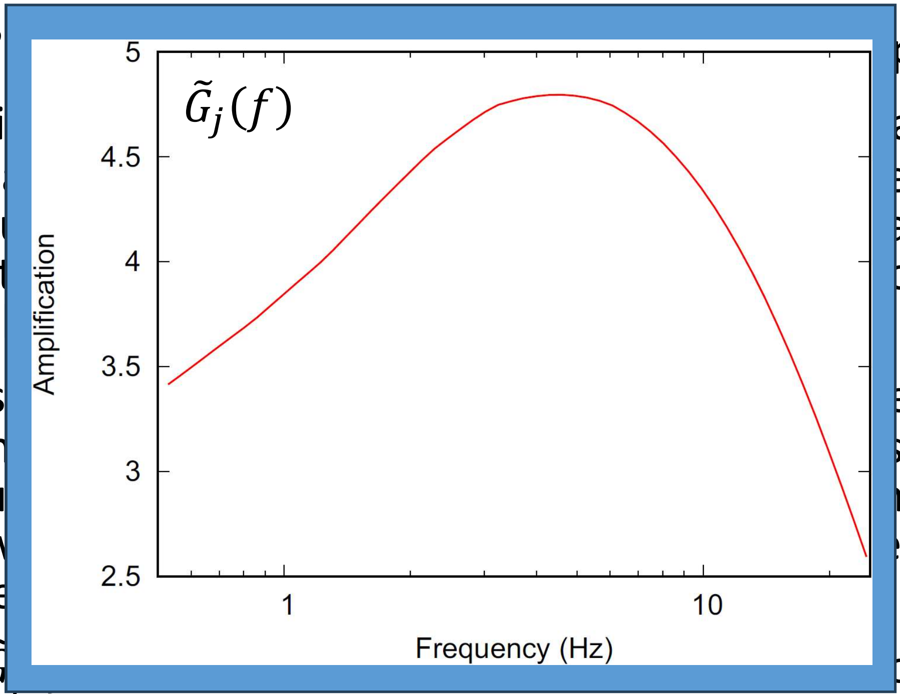
- S_{ij} corresponds to a station-specific S-wave spectrum recorded at a reference rock station located at the reference distance r_{ref}
- To obtain apparent source spectrum $\Omega_{ij}^{GIT}(f)$, one must correct for the residual radiation and wave propagation effects in a half-space, assuming:

$$S_{ij}(f) = \frac{F_{ij}^{S}}{4\pi\rho\beta^3 r_{ref}} \tilde{G}_j(f) \Omega_{ij}^{GIT}(f)$$

Data

$$S_{ij}(f) = \frac{F_{ij}^S}{4\pi\rho\beta^3 r_{ref}} \tilde{G}_j(f)\Omega_{ij}^{GIT}(f)$$

- ρ=3 g/cm^3, β peed at the source.
- F_{ij}^S is the radi ation of horizontal components er a residual effect of random cr age value of the radiation patt of the source mechanisms
- $\tilde{G}_j(f)$ is the s ence rock site (even rock sites ger spect to the homogeneou d the impedance contrast betw and velocity-density at the

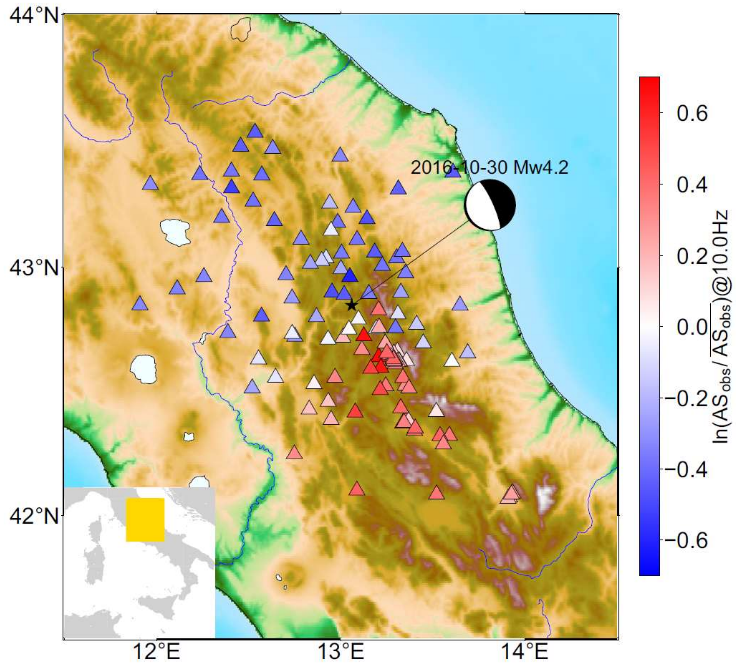


- We assume $\tilde{G}$ or of 2), Boore and Joyner (1997) generic rock site amplification, and shallow attenuation in the form of the kappa effect with κ=0.015 s.

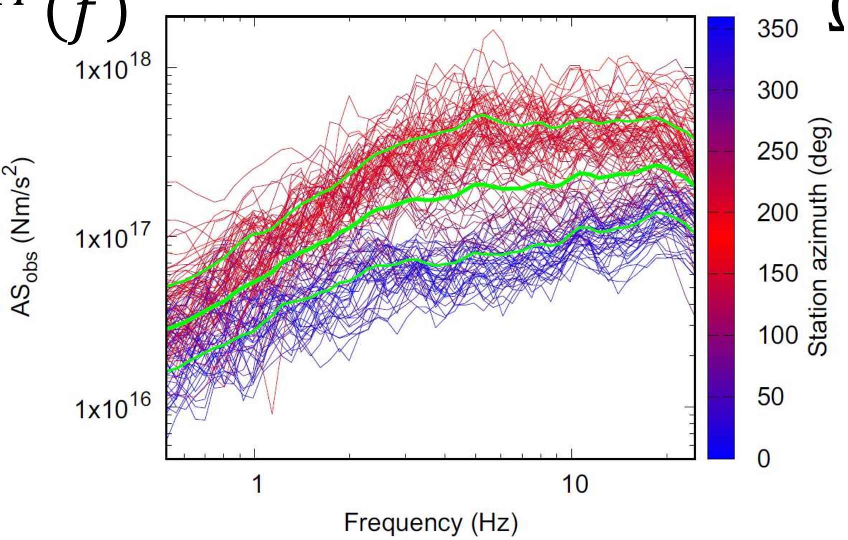
Data

Directive event

2016-10-30 Mw4.2

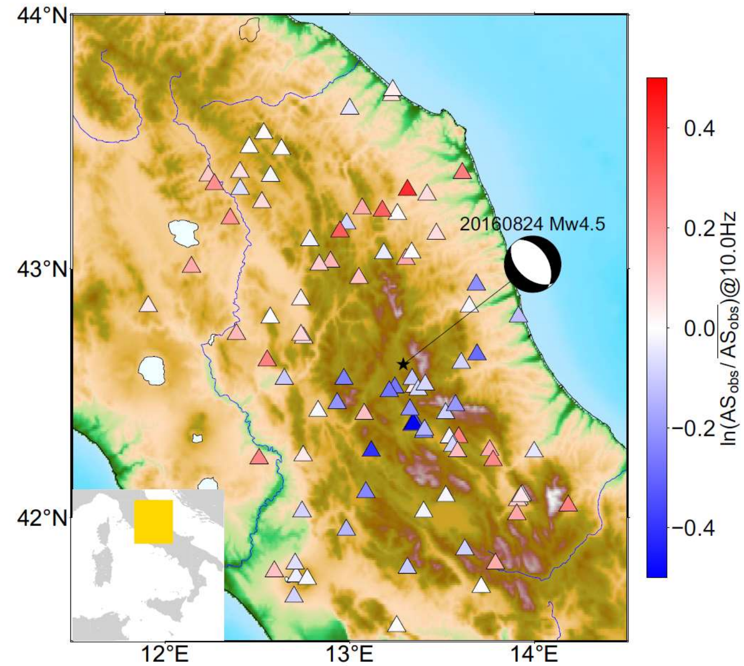


$\ddot{\Omega}_{ij}^{GIT}(f)$

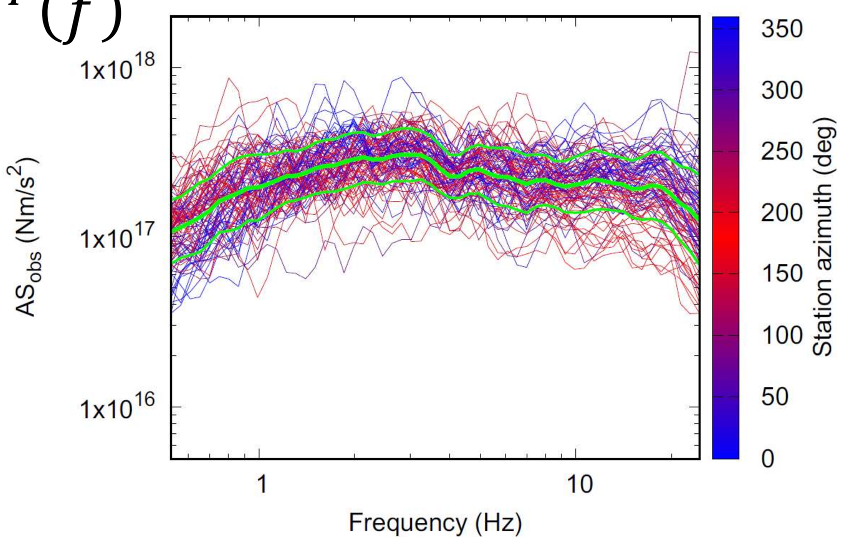


Nondirective event

2016-08-24 Mw4.5



$\ddot{\Omega}_{ij}^{GIT}(f)$



Method (summary)

- We assume slip-weakening friction law with spatially variable dynamic parameters along a fault (2.5x2.5 km^2) embedded in a homogeneous space with constant normal stress (100 MPa)
- Dynamic rupture simulation is performed using FD3D_TSN (Premus et al., 2020) with a 10m grid size
- Synthetic apparent source time functions are calculated from slip rates $\Delta\dot{u}(\boldsymbol{\xi}, t)$ as

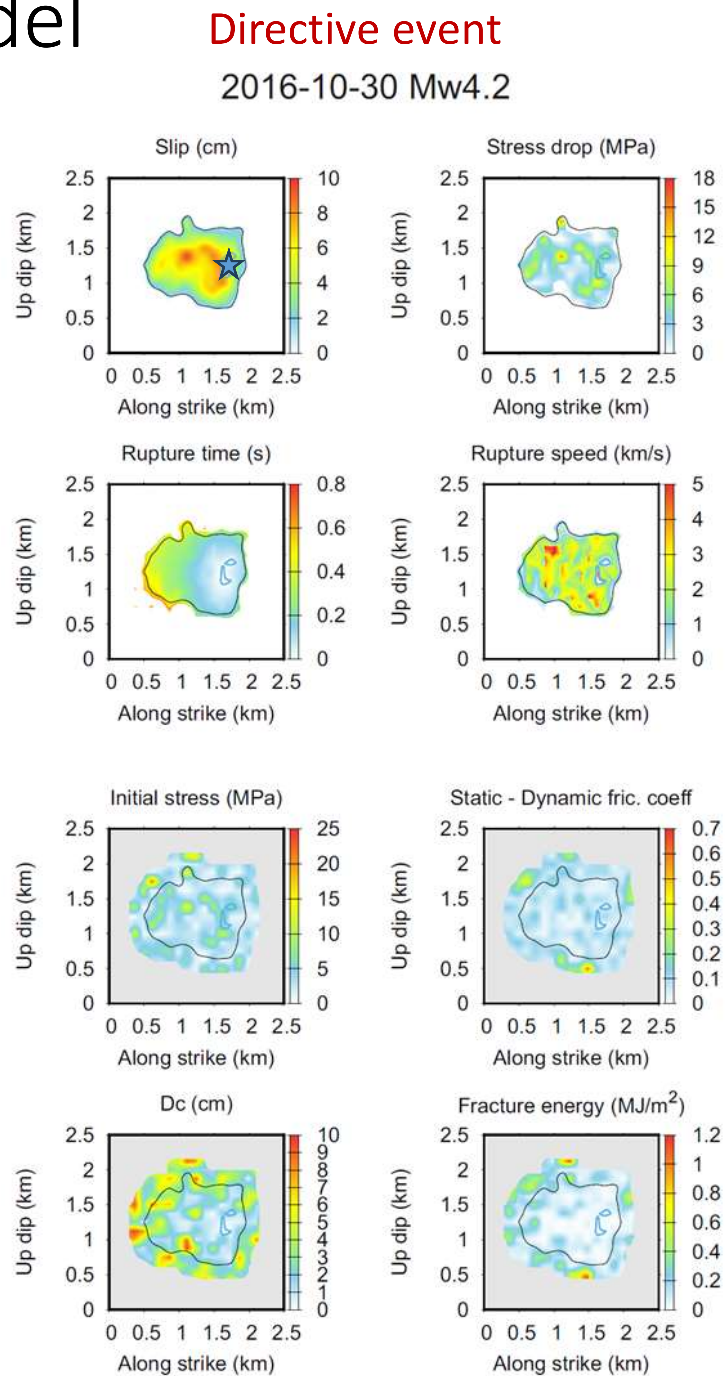
$$\Psi_{ij}^{synt}(t, \boldsymbol{x}_j) = \int_S \mu\Delta\dot{u}\left(\boldsymbol{\xi}, t - \frac{|\boldsymbol{x}_j - \boldsymbol{\xi}|}{\beta}\right) d\boldsymbol{\xi}$$

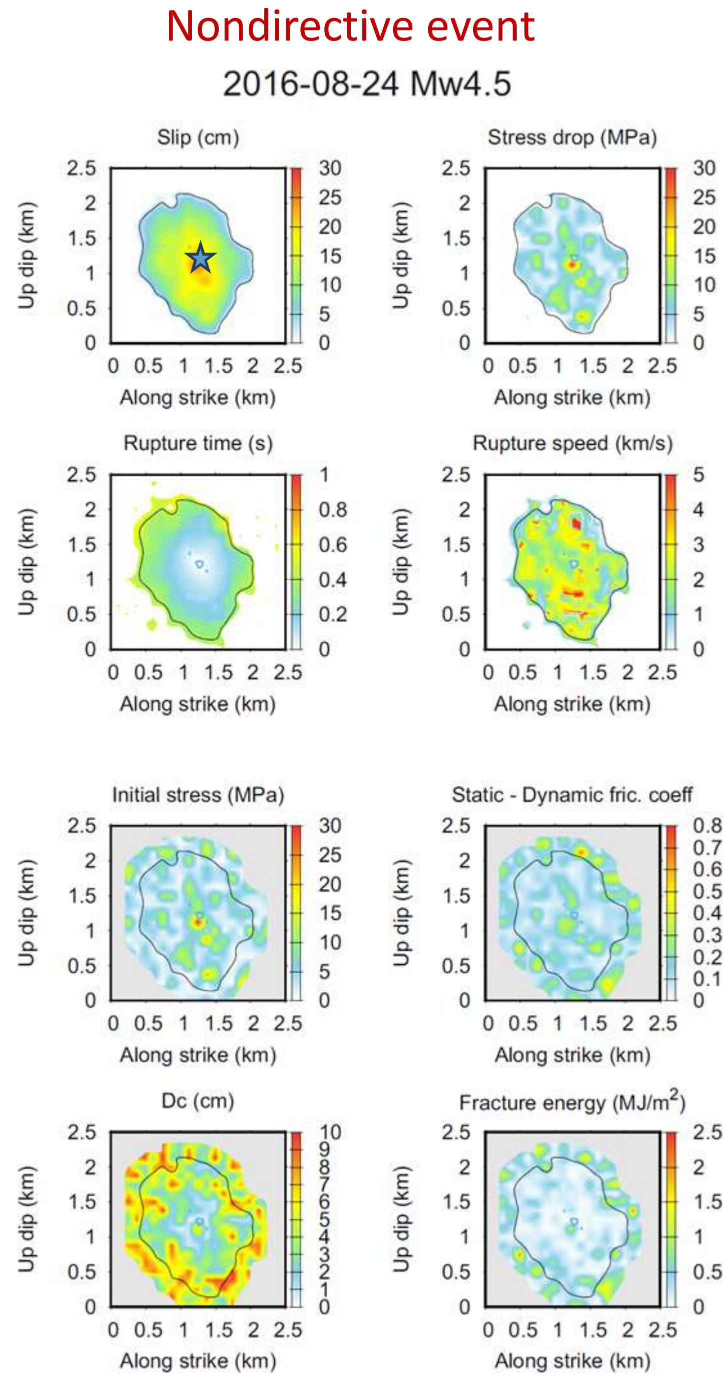
- And the apparent source spectra are their absolute Fourier spectra:

$$\Omega_{ij}^{synt}(f) = \left|\mathcal{F}\left(\ddot{\Psi}_{ij}^{synt}\right)\right|$$

- Parameters are sought using a Markov chain Monte Carlo (McMC) method (parallel tempering) in a Bayesian formulation of the inverse problem (e.g., Gallovič et al., 2019) with constrained seismic moment
- The posterior PDF is sampled in 6 parallel jobs on 3 GPU cards with 48 chains. Each chain started from a randomly altered initial model. We save every 10th step and discard 20% of the initial steps (burn-in phase). This results in ensembles of 4745 and 3667 rupture models for the Mw4.2 and Mw4.5 events, respectively.

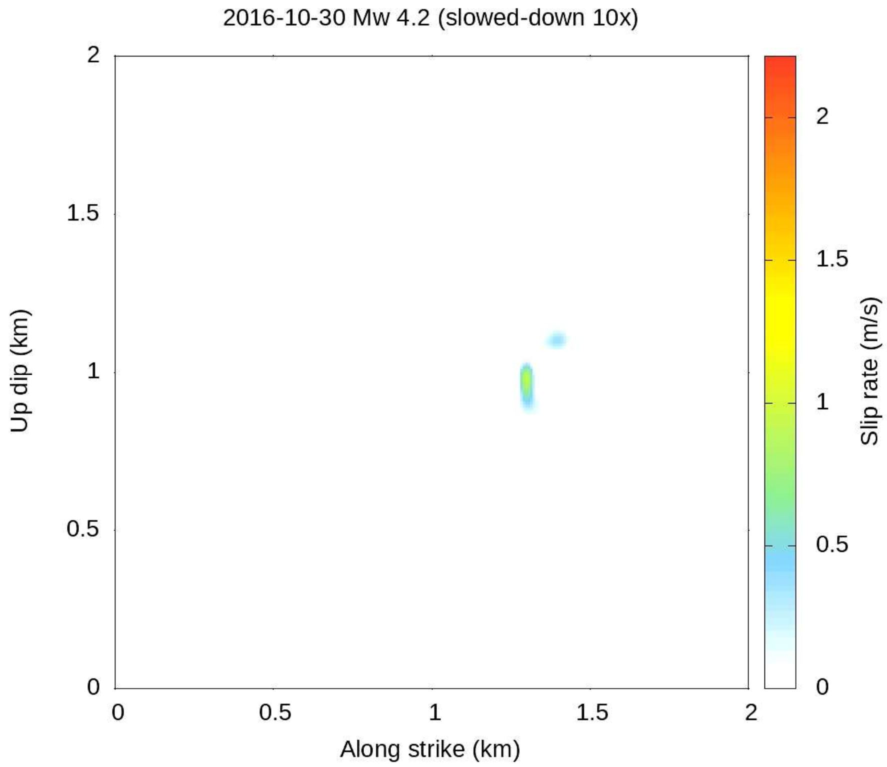
MAP model


Directive event
2016-10-30 Mw4.2
Slip (cm)
Stress drop (MPa)
Rupture time (s)
Rupture speed (km/s)
Initial stress (MPa)
Static - Dynamic fric. coeff
Dc (cm)
Fracture energy (MJ/m2)
Up dip (km)
Along strike (km)

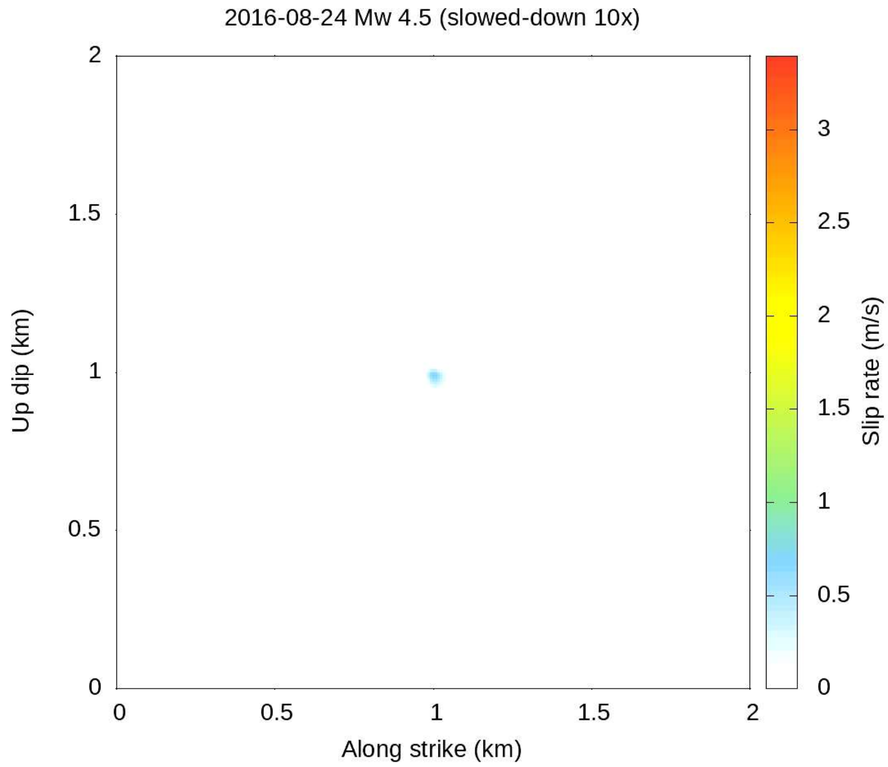

Nondirective event
2016-08-24 Mw4.5
Slip (cm)
Stress drop (MPa)
Rupture time (s)
Rupture speed (km/s)
Initial stress (MPa)
Static - Dynamic fric. coeff
Dc (cm)
Fracture energy (MJ/m2)
Up dip (km)
Along strike (km)

MAP model

Directive event



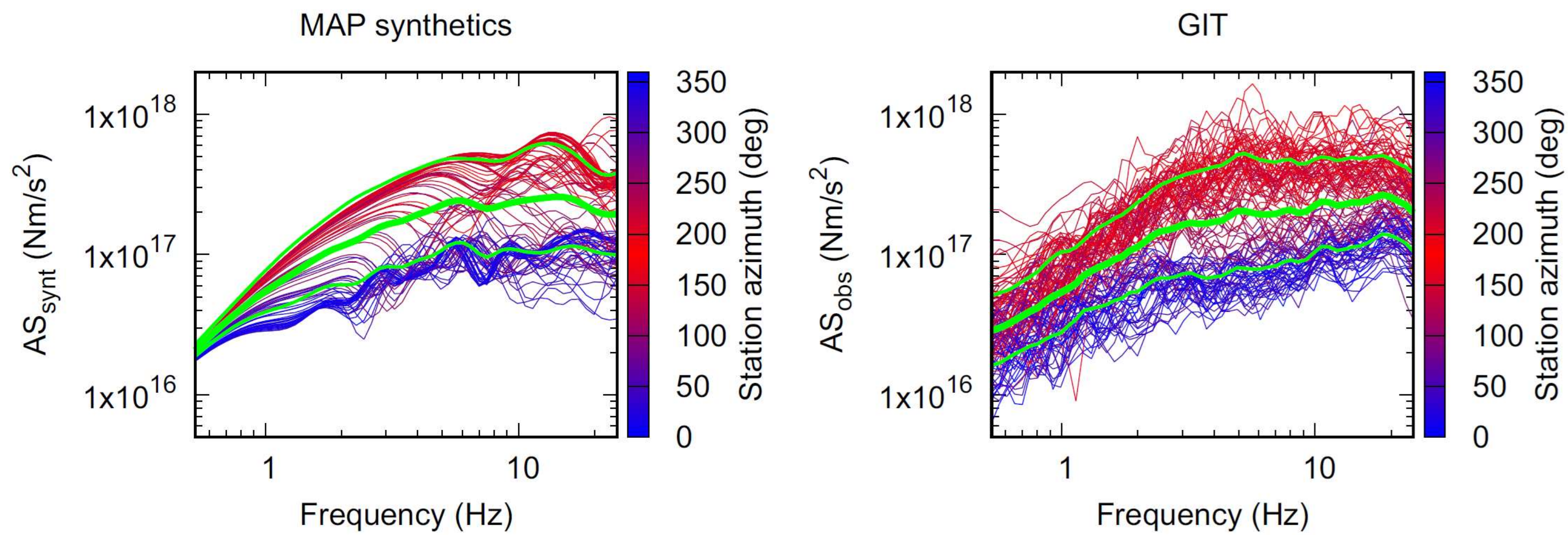
Nondirective event



MAP model

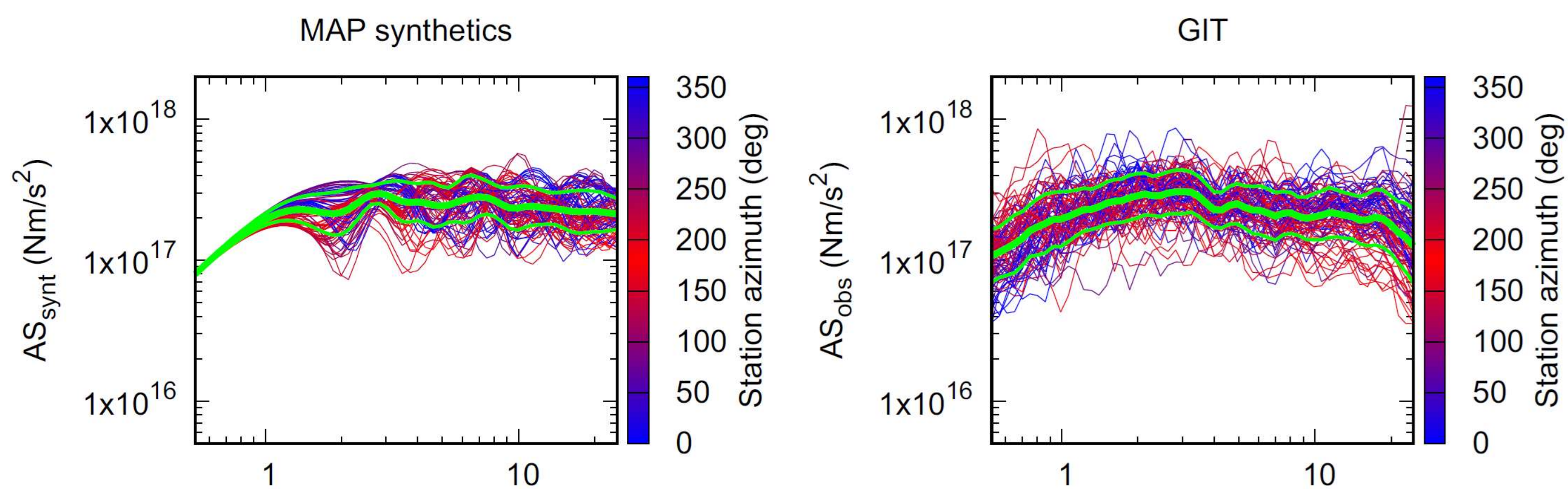
2016-10-30 Mw4.2

Directive event



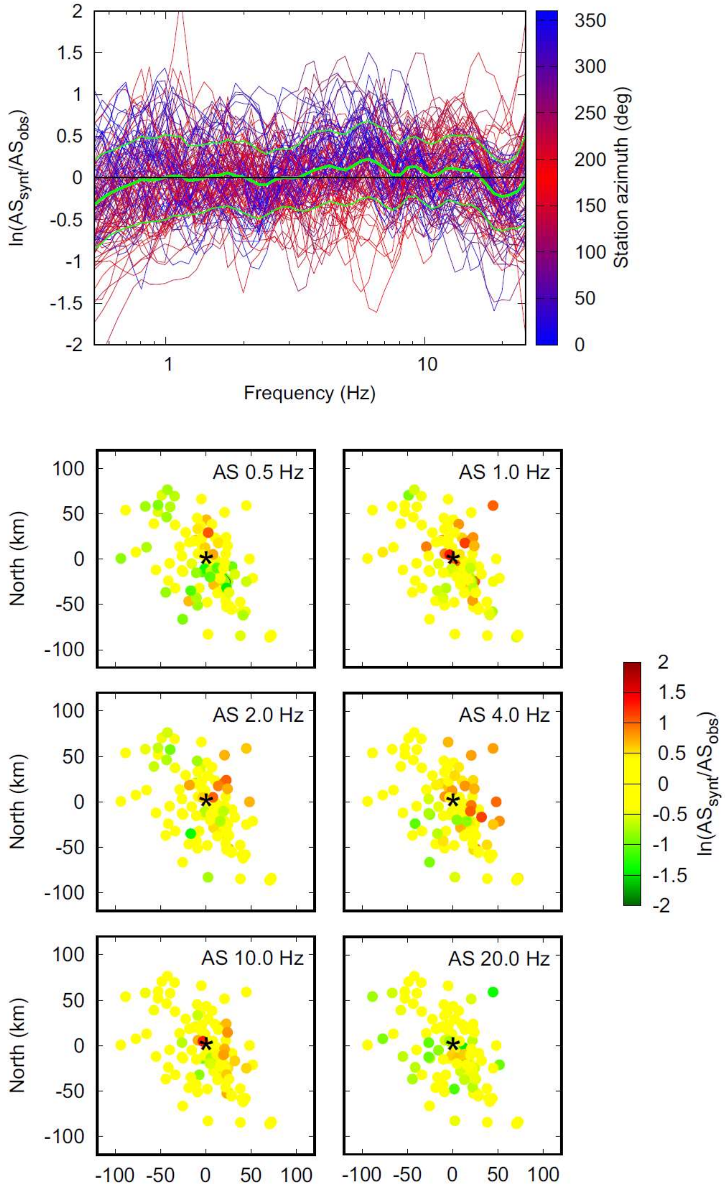
2016-08-24 Mw4.5

Nondirective event

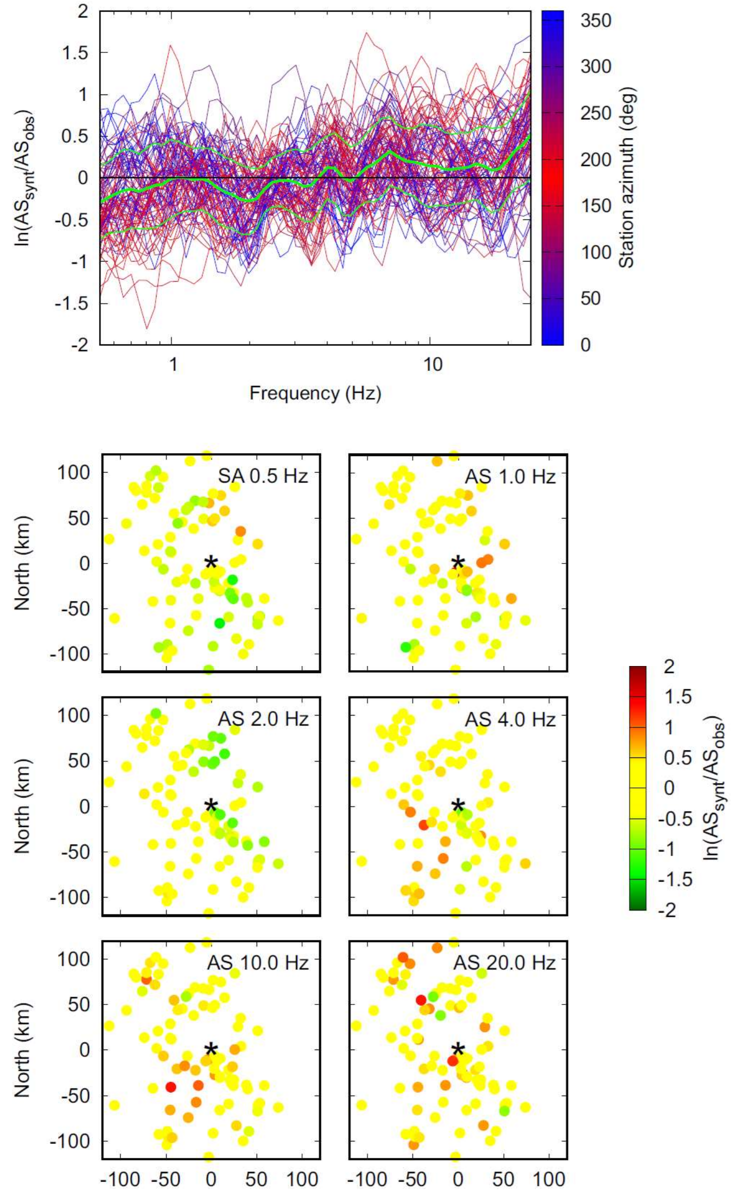


MAP model

2016-10-30 Mw4.2
Directive event



2016-08-24 Mw4.5
Nondirective event



Ensemble rupture properties

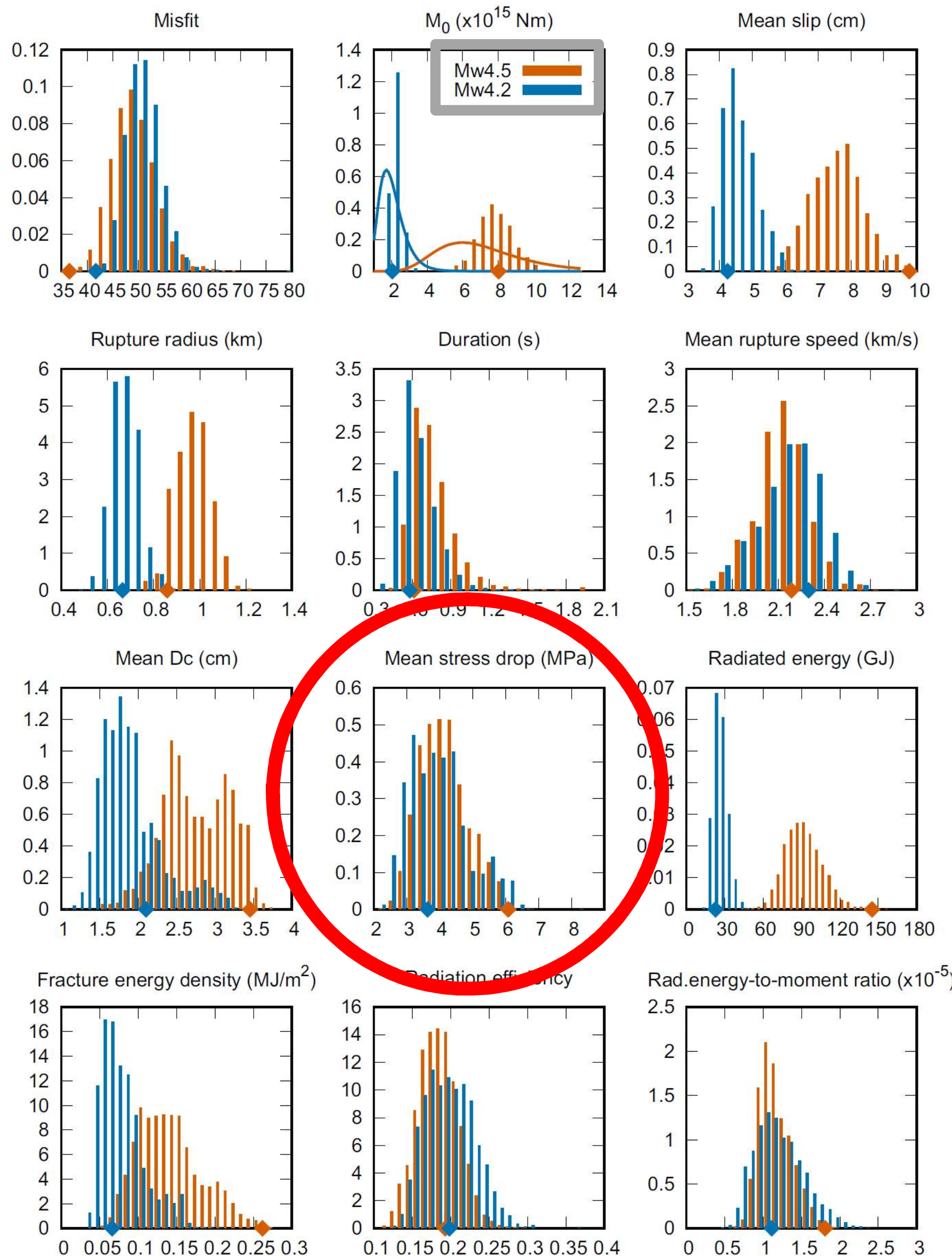
The Mw4.2 and Mw4.5 earthquakes have similar stress drop, rupture speed, stress-drop-to-magnitude ratio, and radiation efficiency.

Other parameters, such as slip, spatial extent of the rupture, average D_c, etc., seem to follow approximately the constant stress drop scaling.

Nondirective event

Directive event

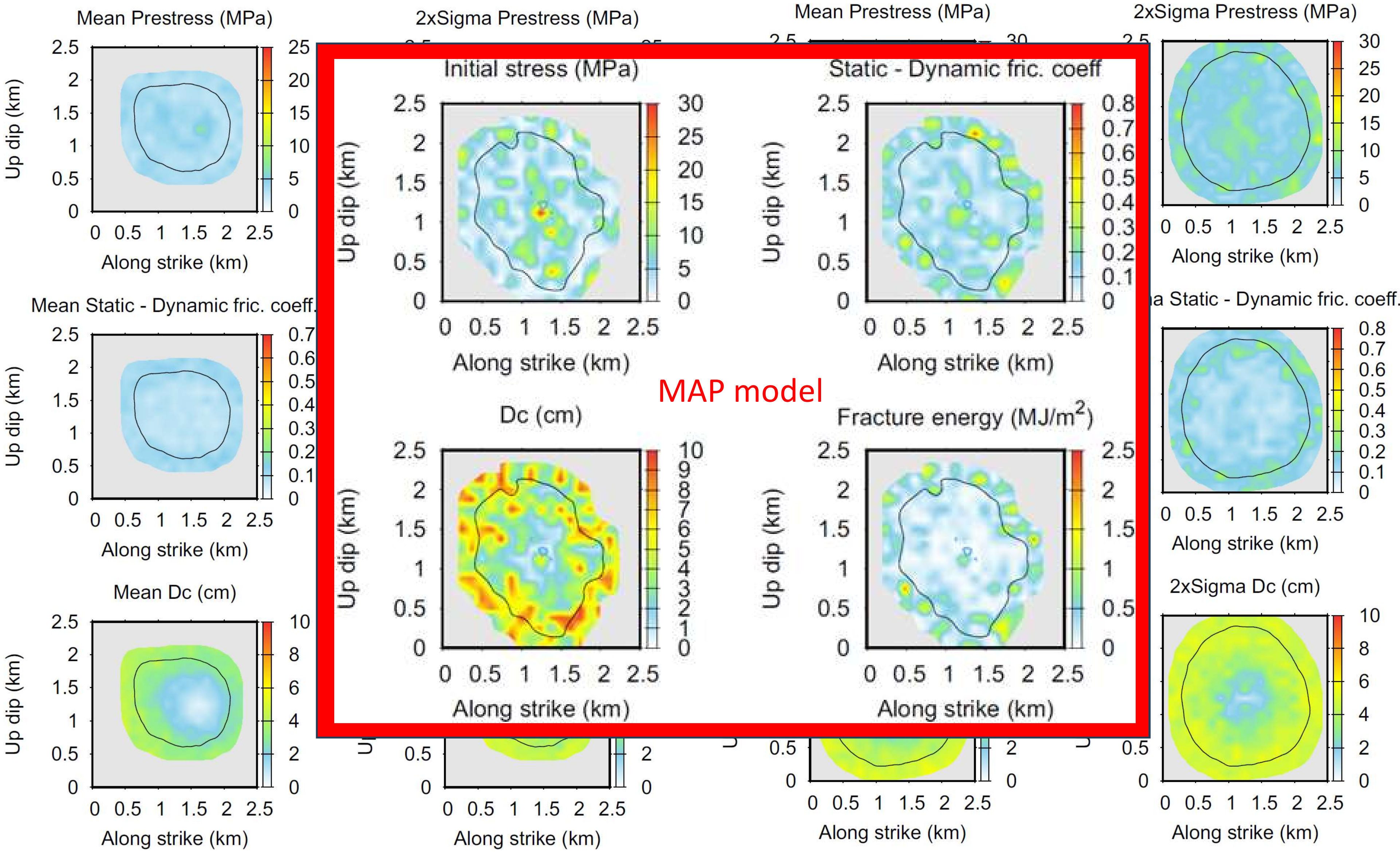
Respective MAP model value



Ensemble rupture properties

Directive event
2016-10-30 Mw4.2

Nondirective event
2016-08-24 Mw4.5



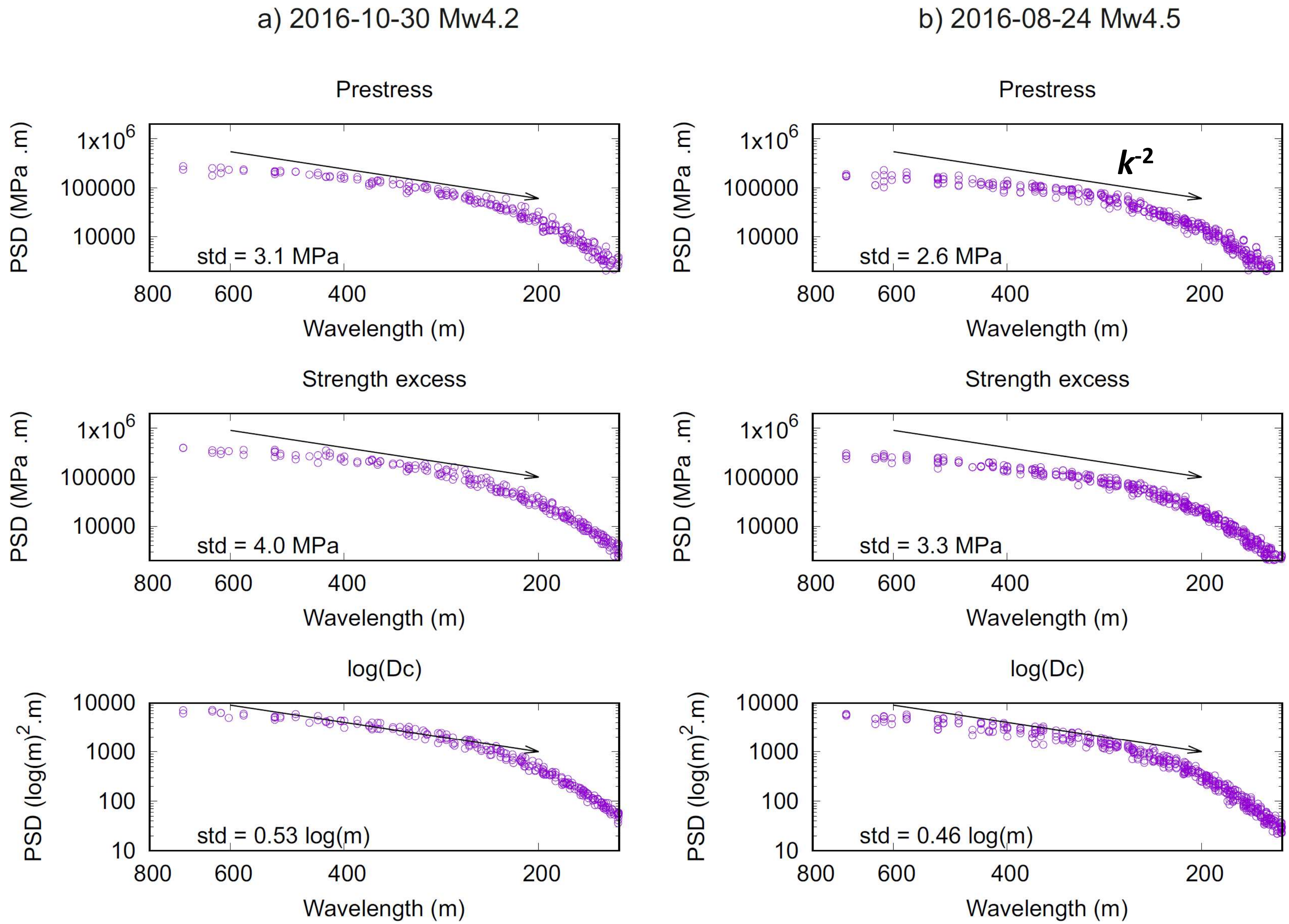
Spectral properties of the dynamic rupture spatial parameters

- Perturbations of a selected dynamic parameter, $g(x, y)$, where x and y are the fault coordinates, are $d(x, y) = g(x, y) - E(g(x, y))$, where E is expectation avaluated by averaging over the model ensemble
- Power spectral density (PSD) $P(k_x, k_y)$ can be calculated as

$$P(k_x, k_y) = \frac{1}{S} E(\mathcal{F}[d(x, y)s(x, y)]^2)$$

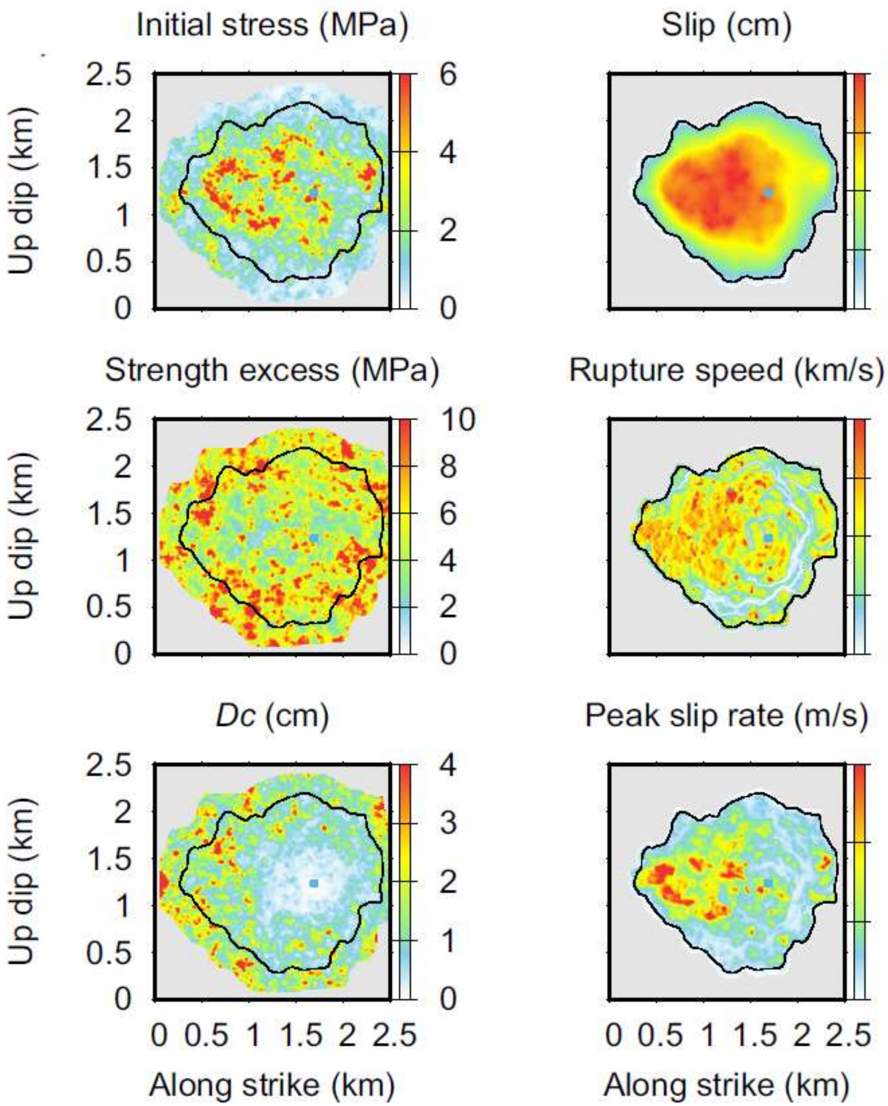
- Where S is the fault area, and $s(x, y)$ is the slip distribution $\Delta u(x, y)$ normalized by the square root of its average power, $s(x, y) = \Delta u(x, y) / \sqrt{\int \Delta u(x, y)^2 dS / S}$.

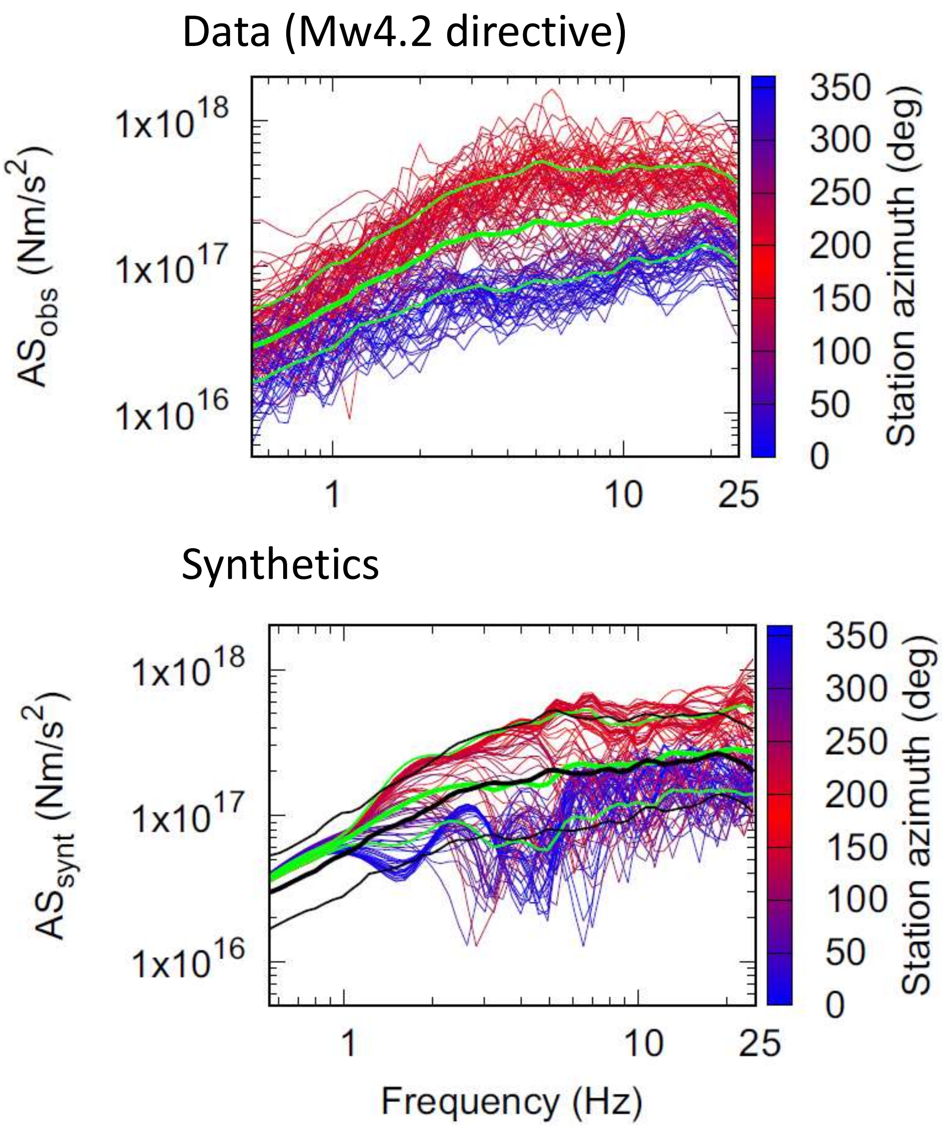
Spectral properties of the dynamic rupture spatial parameters



Can constrain dynamic parameter heterogeneity for rupture scenarios (under testing by Lav Joshi)

Models with k^{-1} random perturbations in dynamic parameters (future scenario modeling)





Joshi, Gallovič, Sgobba (CommEE, just published)

Summary and outlook

- It is challenging to constrain kinematic rupture parameters
- Possible approaches are:
  - posterior comparison with GMPEs
  - incorporation of physics-based dynamic models constrained by dynamic inversions with added small-scale heterogeneities

- Pros and cons of the dynamic rupture approach:
  - The employed physical constraint (elastodynamic equation and friction law) effectively reduces the uncertainty of kinematic rupture models
  - Very good data coverage (seismic and geodetic) is necessary
  - Computationally still rather prohibitive for routine applications
  - How do the results depend on the choice of the (laboratory-derived) friction law?

- Generalization of the results of dynamic earthquake source inversions will enable better constrained physics-based

Thank you for your attention…