

Modeling Seismic Source Directivity Effects from Apparent Source Spectra: A GIT-Based Approach for Small Earthquakes in Central Italy

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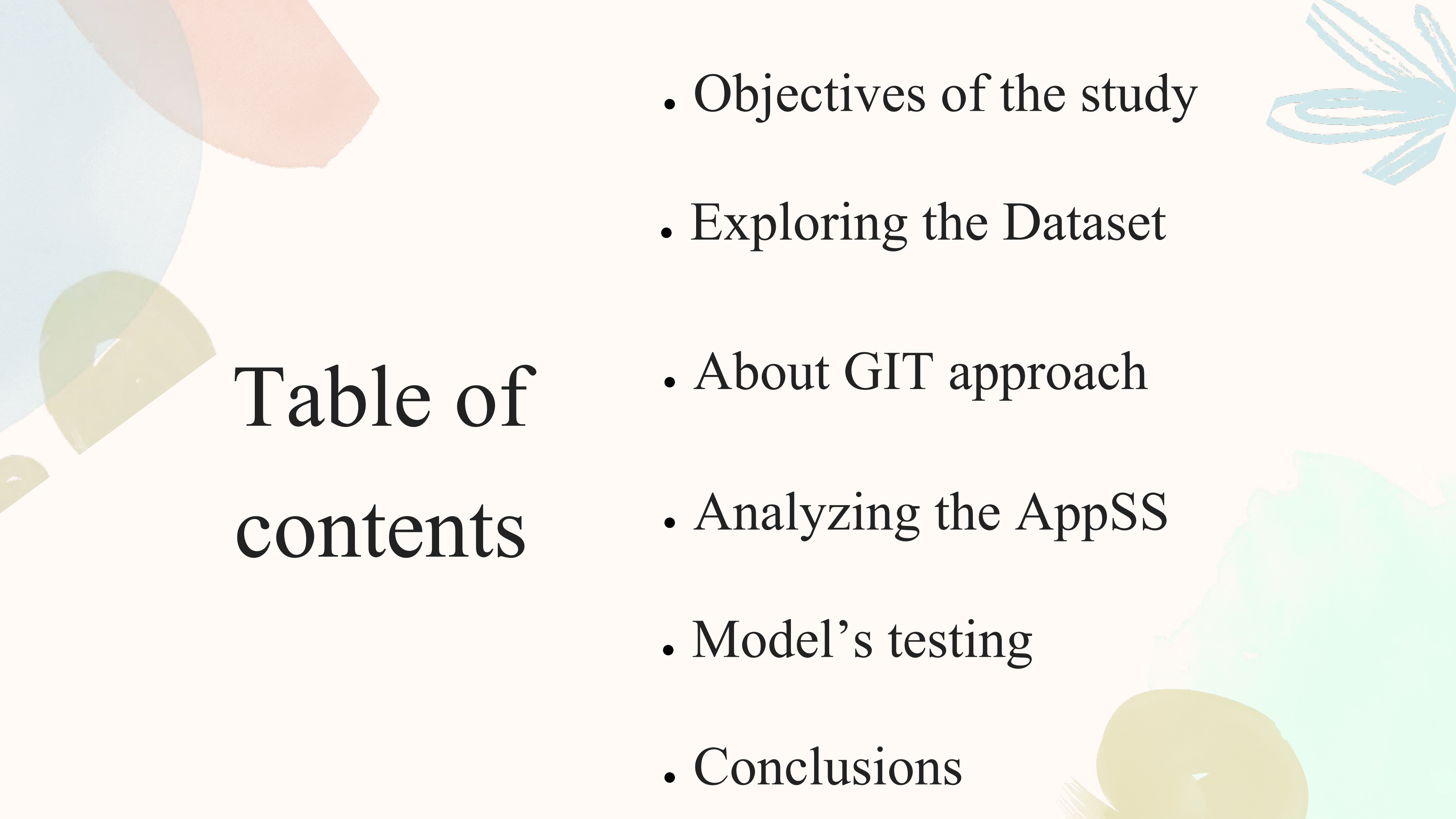


Table of contents

- Objectives of the study
- Exploring the Dataset
- About GIT approach
- Analyzing the AppSS
- Model's testing
- Conclusions

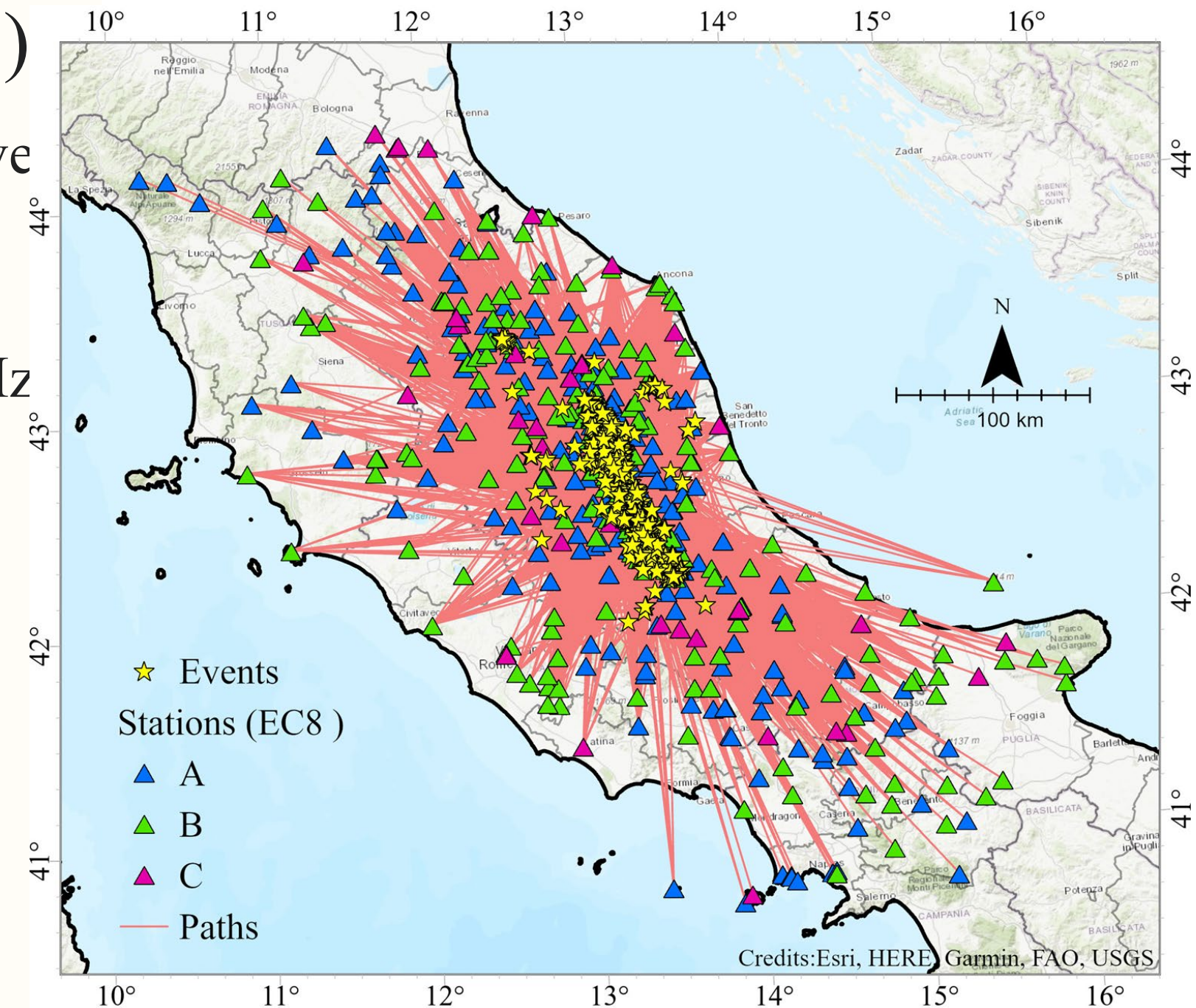
Objective

- **obtain** the Apparent Source Spectra (**AppSS**) with the Generalized Inversion Technique (**GIT**) for the **ITACAext 2.0** dataset
- **analyzing** the **source spectra (AppSS)** to capture empirical **evidence**
/ check for the **presence of directivity**
- **testing** different **models** and **defining** the **best model**, supported by **residual analysis** to check the **accuracy** of the **adopted fitting**

Exploring the Dataset

- Dataset: **ITACAext 2.0** (Lanzano et al., 2025)
 - + **accelerometer** and **velocimeter** recordings of $M_w \geq 3.0$ events
 - + **manually processed** recordings from **1972** to **2022**
 - + ordinates of the **Fourier spectrum** ($f=0.04\text{--}50$ Hz) calculated for the **entire signal** as well as for the **S-phase**
- Working Dataset : **ITACAext 2.0 CI**
 - + Focused region : **Central Italy**
 - + Tectonic regime : **Shallow Active Crustal**

ITACA = Italian ACcelerometric



Distribution of earthquakes (yellow dots), stations (color-coded triangles), and paths (pink lines) in the study region.

About GIT

- the Generalized Inversion Technique (**GIT**) represent an alternative tool for evaluating empirical site responses in the frequency domain (Castro et al., 1990)

$$FAS_{ij}(f) = S_i(f) + P_{ij}(R_{ij}, f) + Z_j(f)$$

GIT input

- $FAS_{ij}(f)$ is the *Fourier Amplitude Spectrum* for each *frequency* f recorded at *event* i at *site* j

GIT output

- $S_i(f)$ is the *source function* for *event* i
- $P_{ij}(R_{ij}, f)$ is the *path* contribution over *distance* R for *event* i at *site* j
- $Z_j(f)$ is the *site-response term* at *site* j

Parameter	Description	Value
Components	Component used for the inversion : "H" or "Z"	“H” (horizontal)
Mean	Calculation method of the two "H" components	“VECTORIAL”
f_min	Min frequency to consider for the analysis	5.0 Hz
f_max	Max frequency to consider for the analysis	25.0 Hz
Bin type	Creates a distance vector composed by a linear distribution of values between "dist_min" and "dist_max" with a step as given in the variable "dist_step"	“LIN” (Linear)
Dist_min	Min distance for type_bin = LIN or LOG	5.0 Km
Dist_max	Max distance for type_bin = LIN or LOG	125.0 Km
Dist_step	Step in km between values (used for type_bin = LIN)	2.0 Km
Ref. distance	Reference distance [km] for which attenuation functions at all frequencies is set to vdref	10.0 Km
vdref	Attenuation amplitude at all frequencies in correspondence of dref	1.0
Weight ref. dist.	Weight of attenuation constraint at reference distance dref	5.0
Reference sites	“IT.LSS.00” “IT.SLO.00” “IT.SNO.00” “IT.NRN.00” “IT.MNF.00” “IT.SDM.00”	
Knot	High_freq decay kappa value [s]; set to Na to skip Knot	0.0
Weight_smo	Weight for smoothing condition on attenuation functions, if dynamic smoothing, max. weight will be WSMOOTH, min can be set in functions themselves, currently on WSMOOTH/4	25

Analyzing the AppSS



- the Apparent Source Spectra (AppSS), i.e., the **frequency-dependent** representation of seismic **source radiation** as observed at a **recording site**

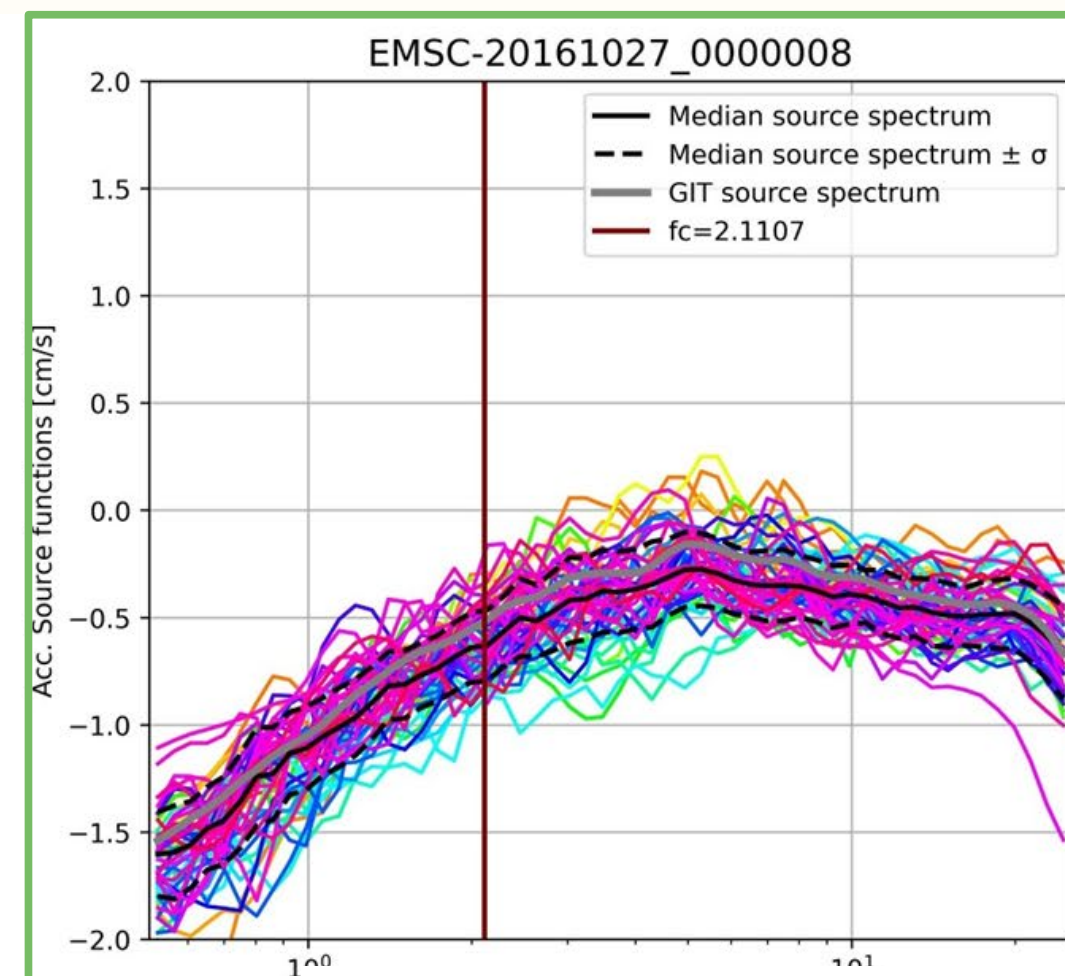
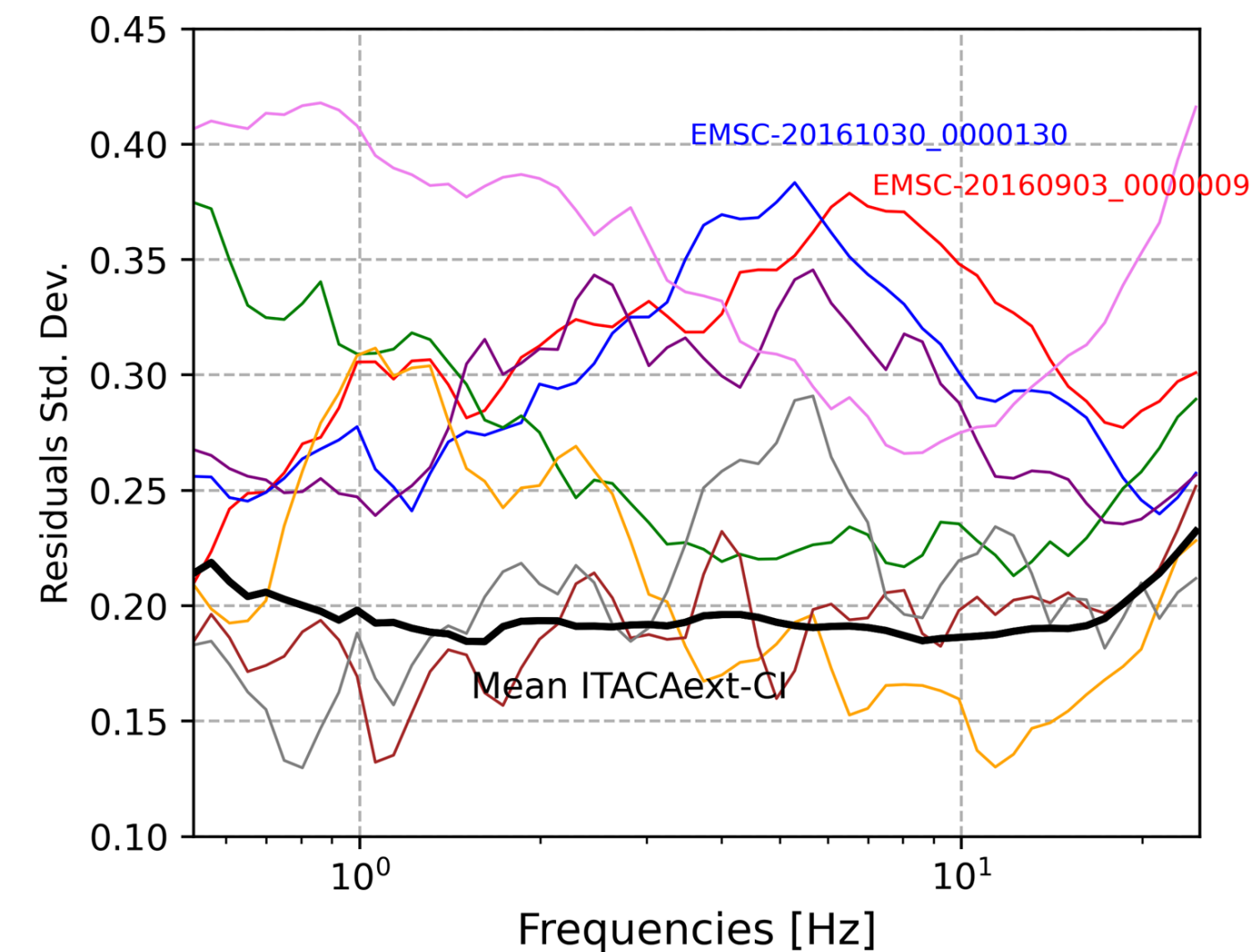
$$\underbrace{S_{ij}(f)}_{\text{AppSS}} = FAS_{ij}(f) - P_{ij}(R_{ij}, f) - Z_j(f)$$

- $FAS_{ij}(f)$ is the *Fourier Amplitude Spectrum* for each *frequency* f recorded at *event* i at *site* j
- $P_{ij}(R_{ij}, f)$ is the *path* contribution over *distance* R for *event* i at *site* j
- $Z_j(f)$ is the *site-response term* at *site* j

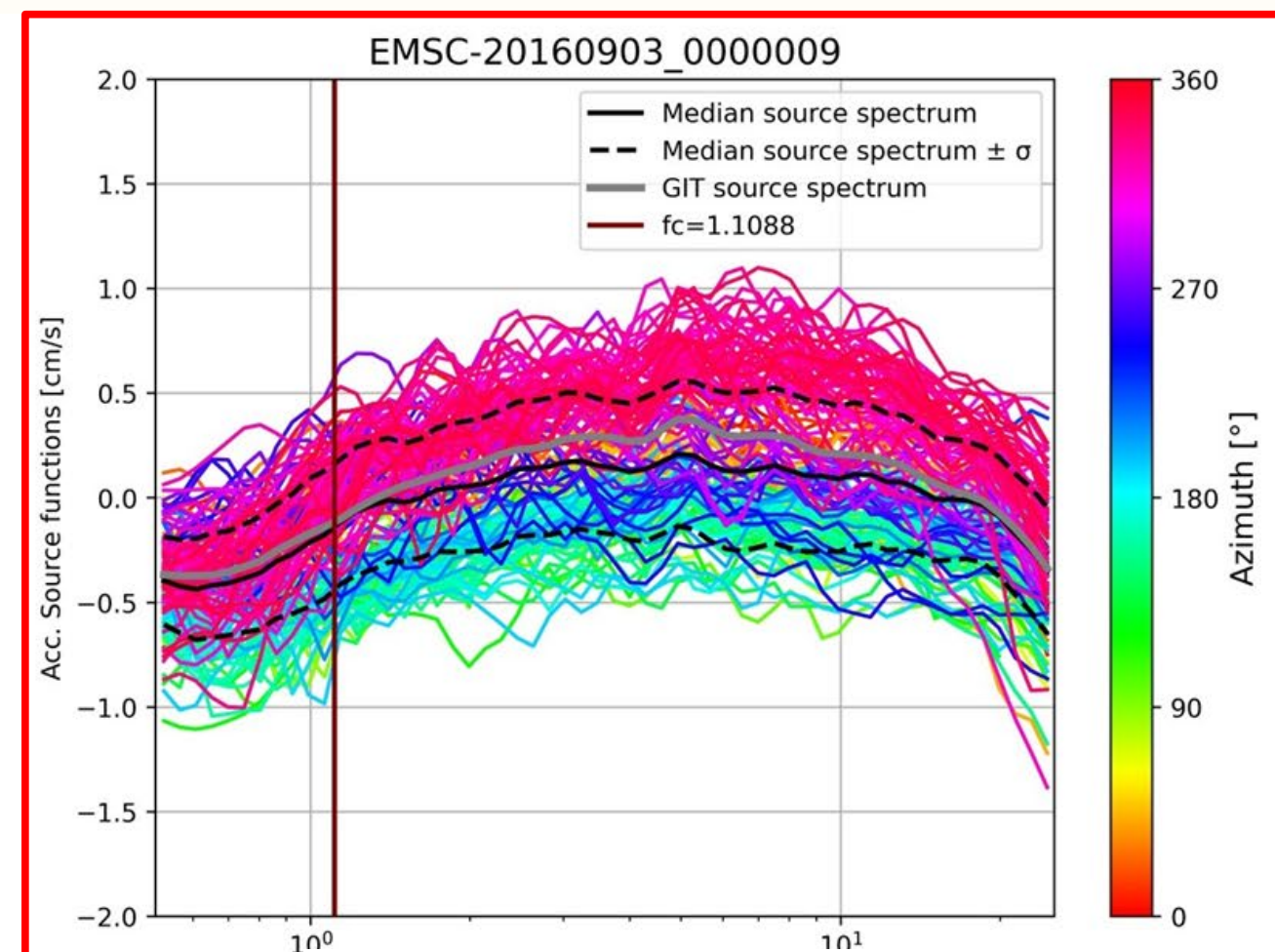


Analyzing the AppSS

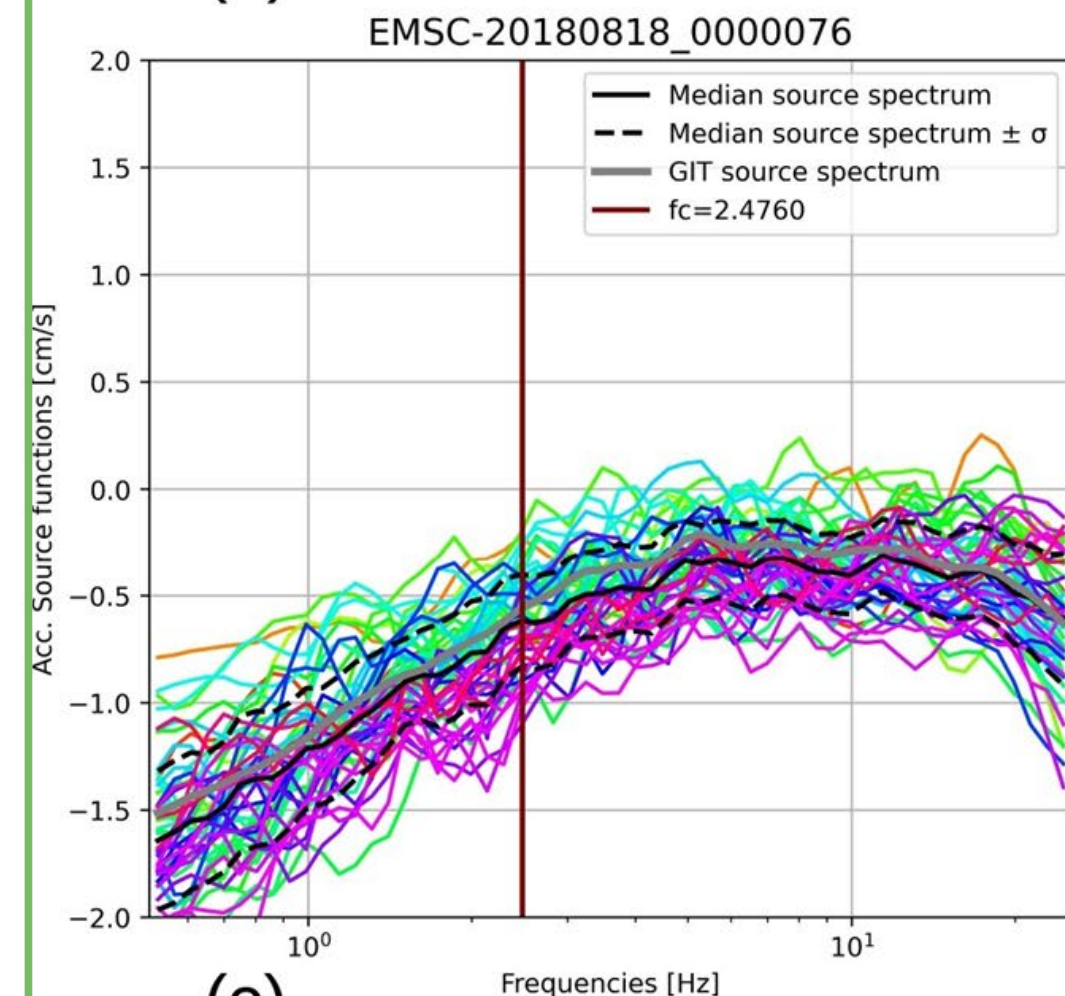
- identifying directivity events e.g., AppSS shape or residuals standard derivation



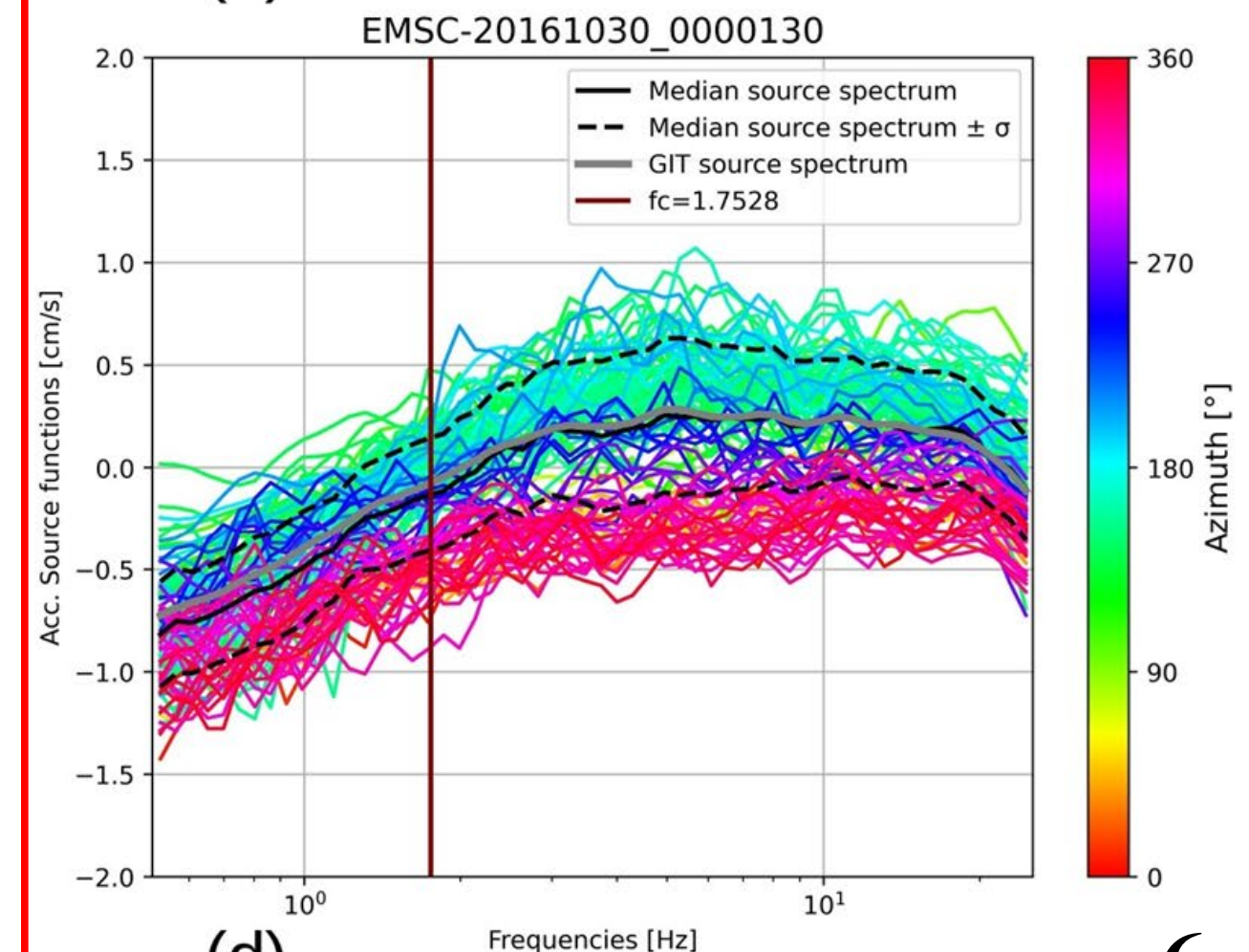
(a) *No Directivity Effects*



(b) *Strong Directivity Effects*

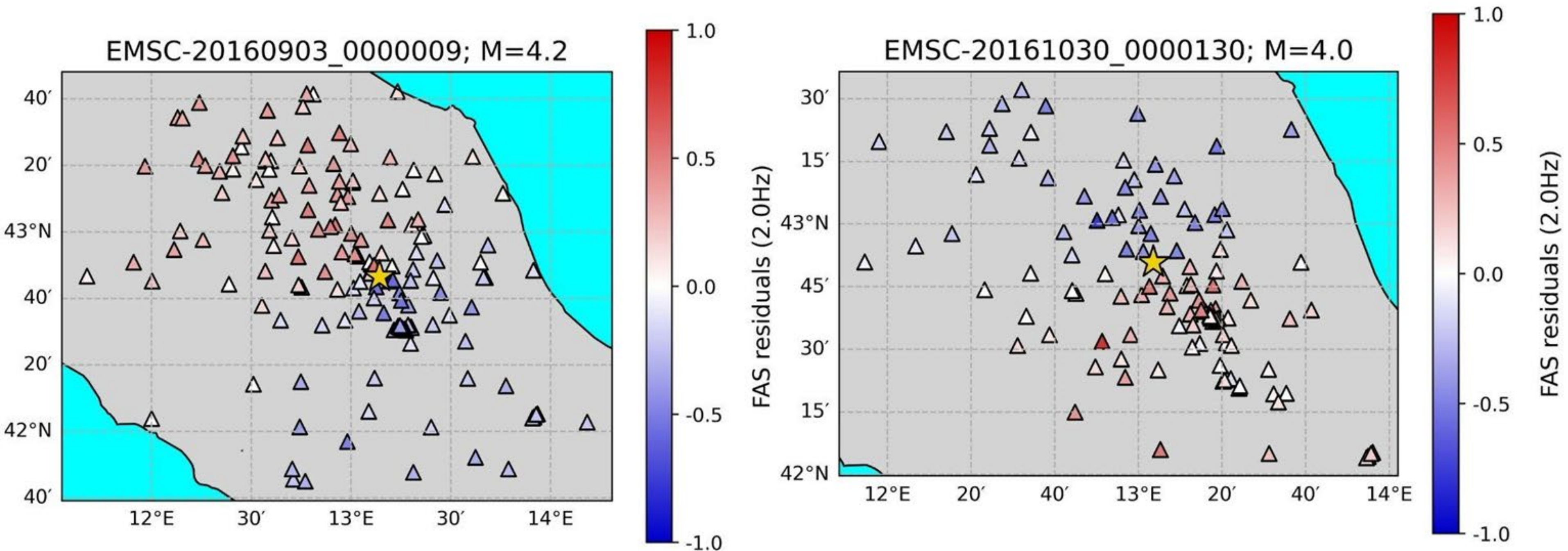


(c)



(d)

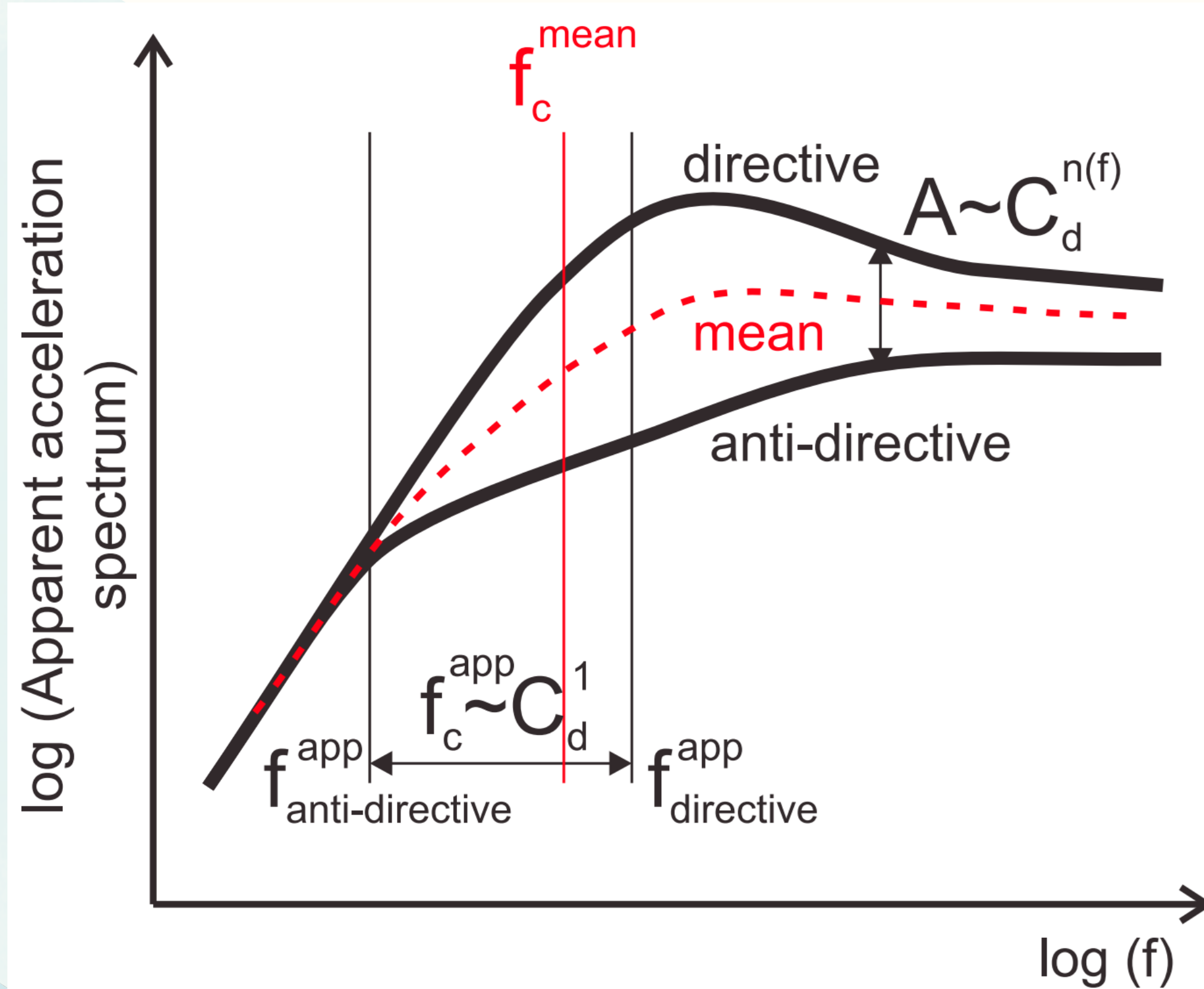
Spatial distribution of AppSS amplitudes



Azimuthal variations in AppSS amplitudes (normalized to their 2.0 Hz means)

Note: For the **directivity analysis** was selected a subset of events with **optimal azimuthal coverage-events recorded by at least 40 stations**, with more than **4 stations** in each quadrant **[0°-90°-180°-270°]**.

Model's Tests

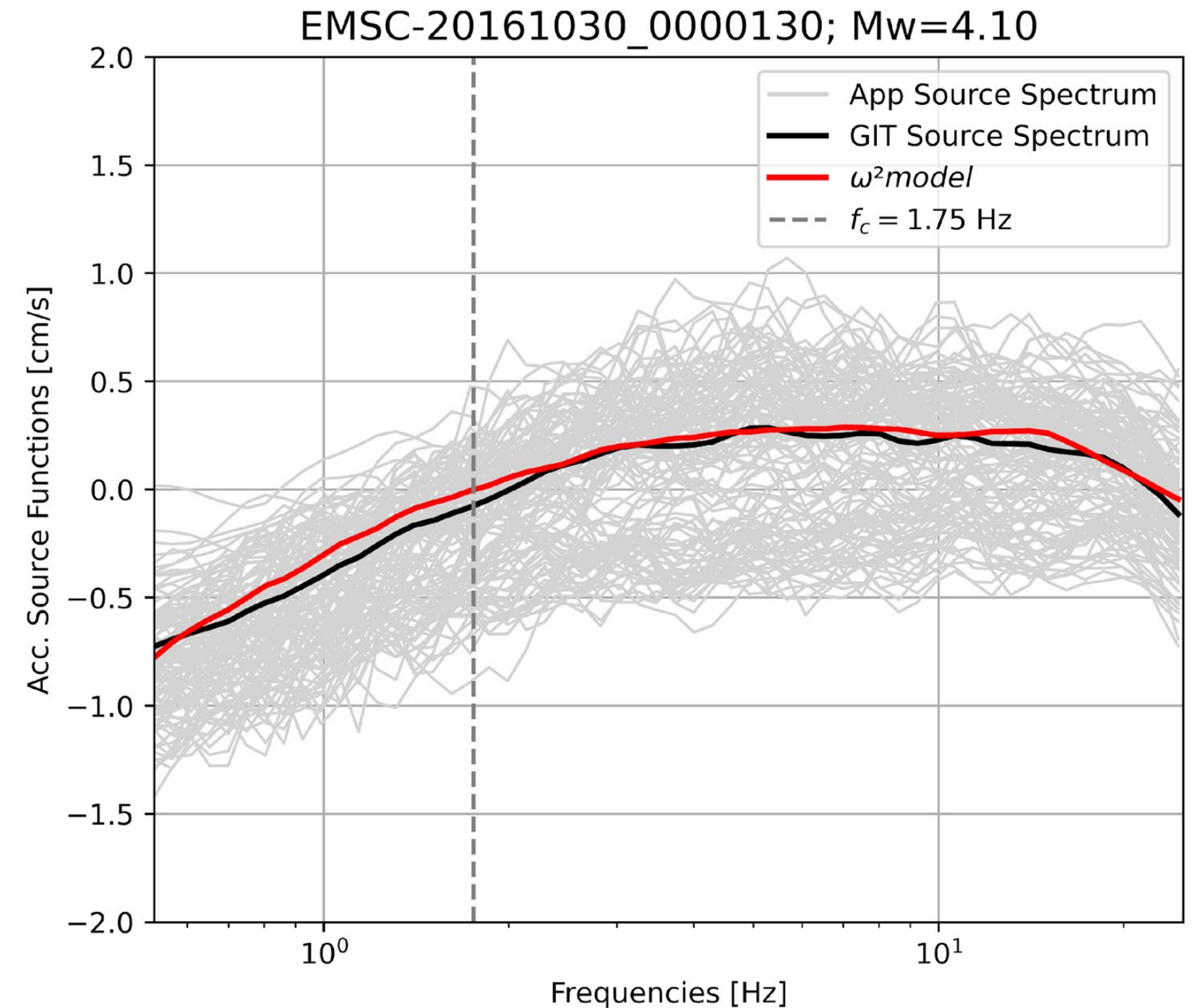
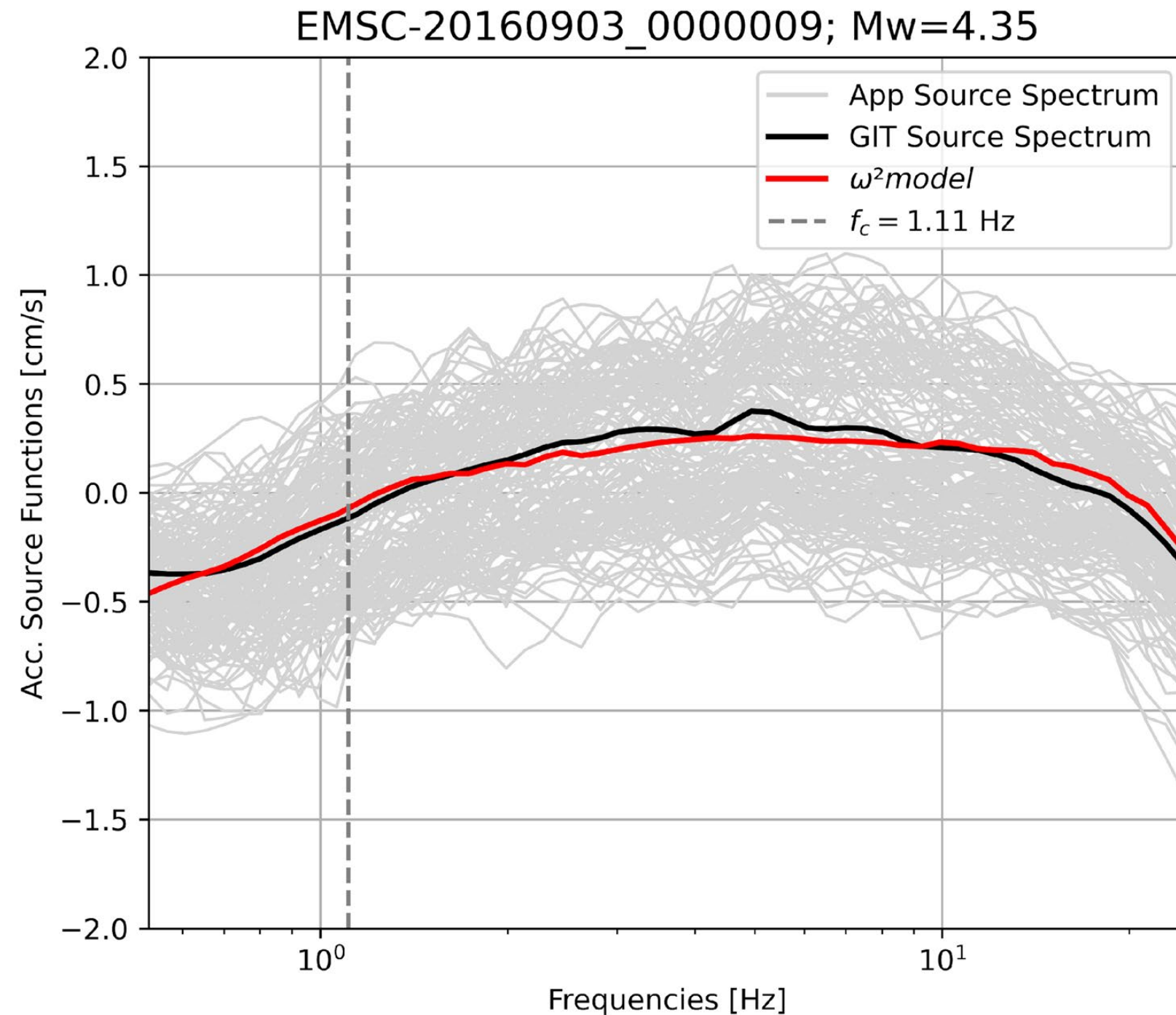


Schematic illustration of apparent source spectra described by the generalized Ben-Menahem [1961] directivity coefficient C_n with generally frequency dependent n as suggested by advanced theoretical models (retrieved from Pacor et al., 2016)

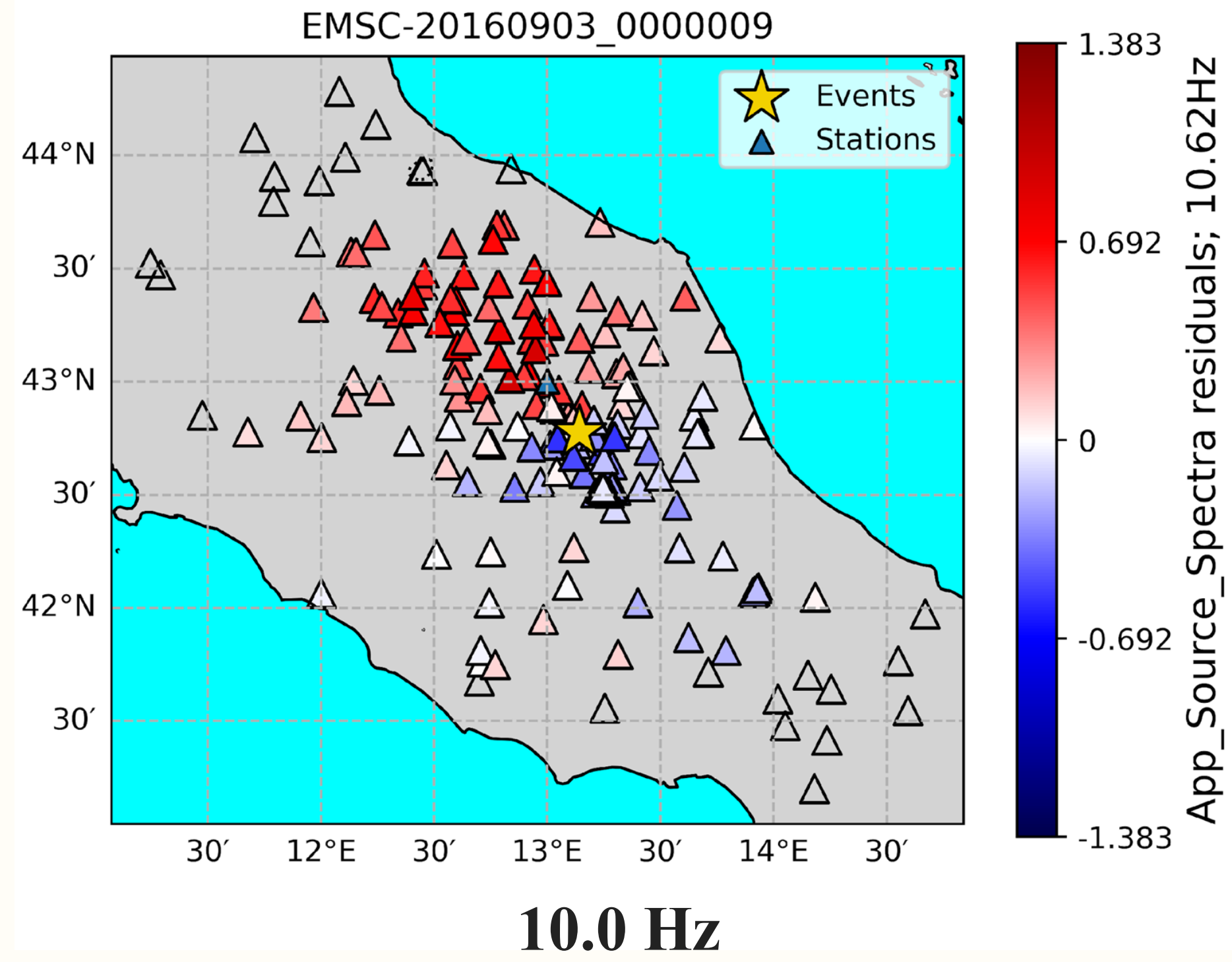
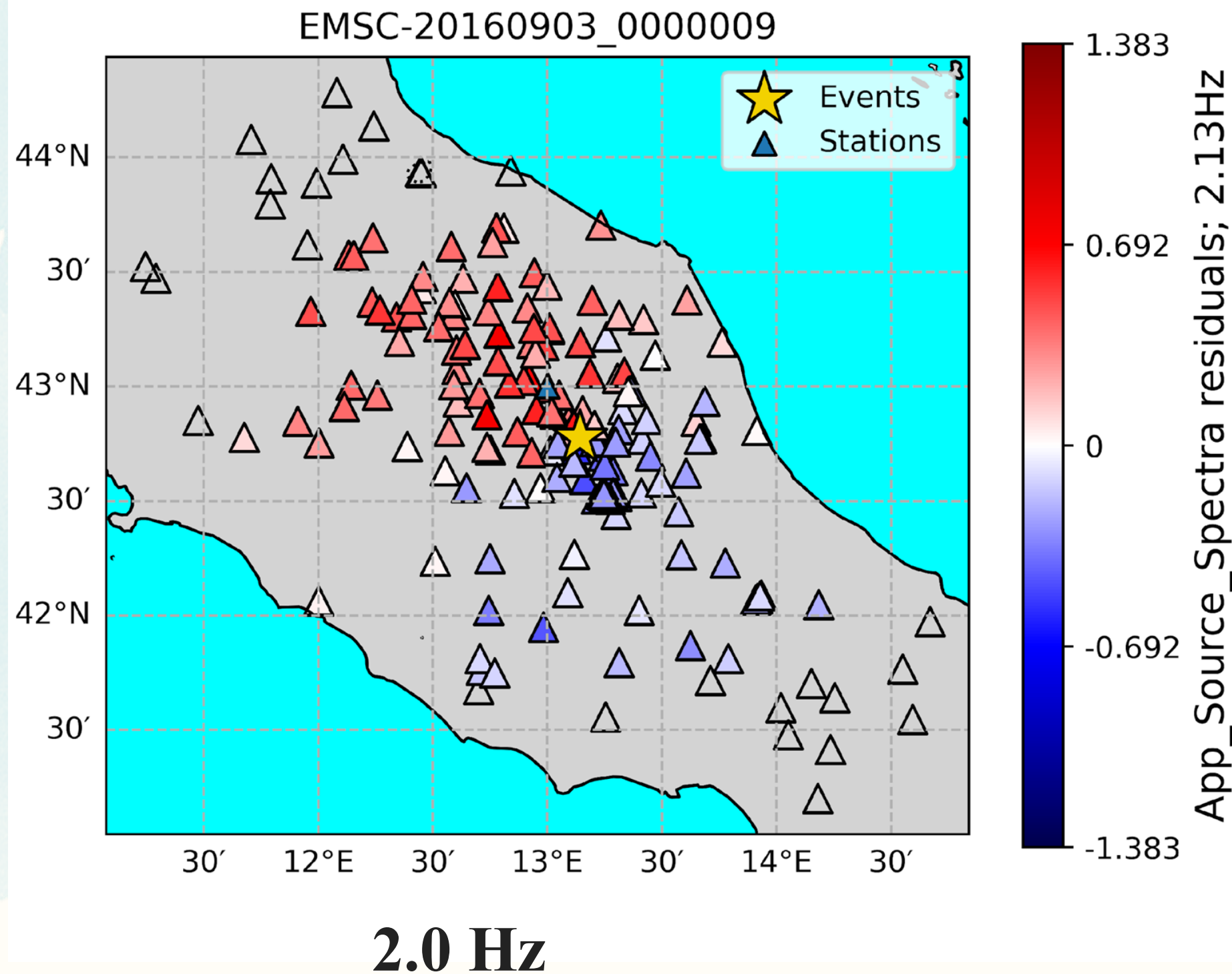
ω^2 model (Brune, 1970)

- *Step-1*: fitting the source spectra using an ω^2 model (Brune, 1970) to

estimate a constant value of the seismic moment (M_0) and corner frequency



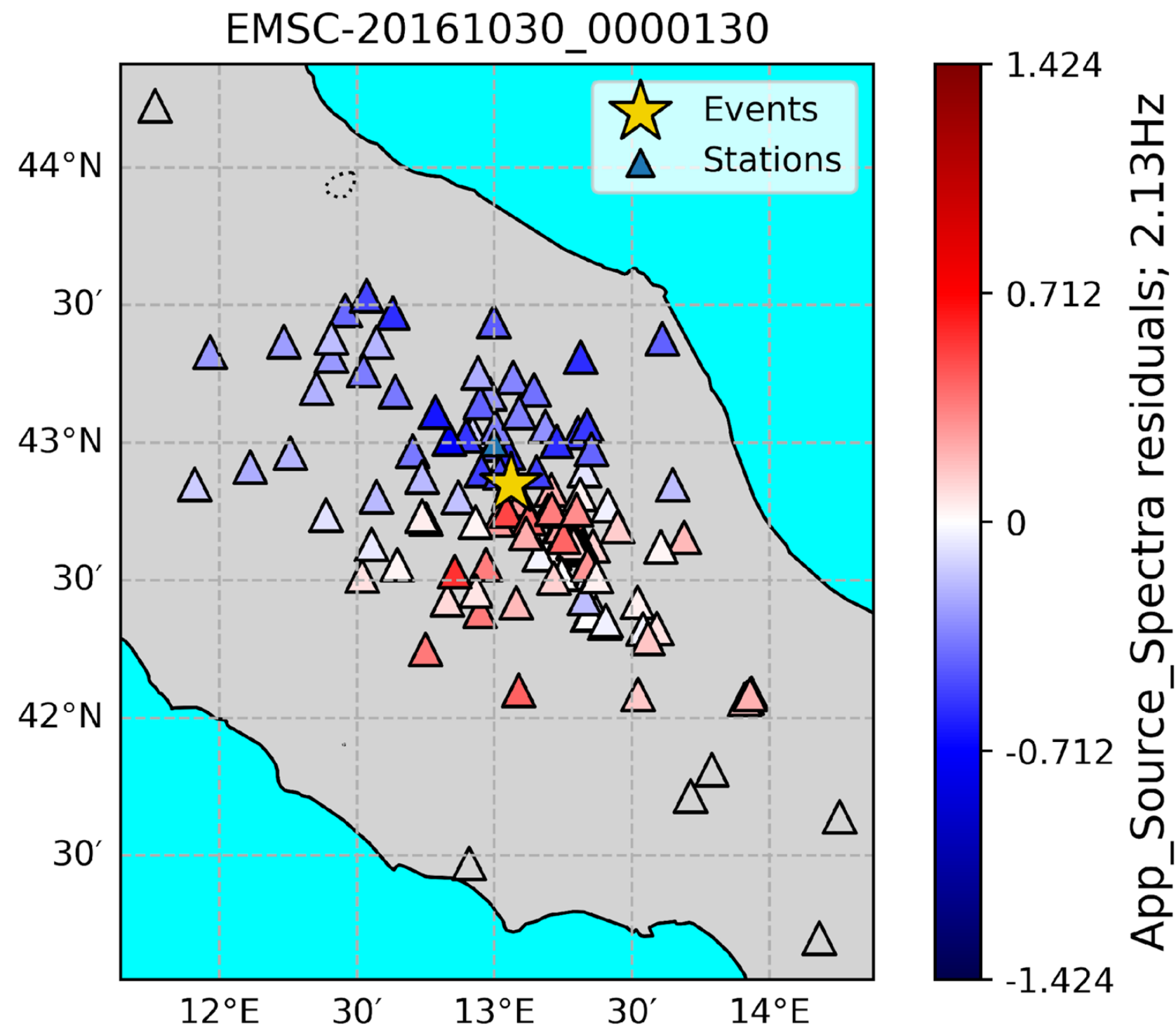
Spatial Variations of ω^2 model (Brune, 1970)



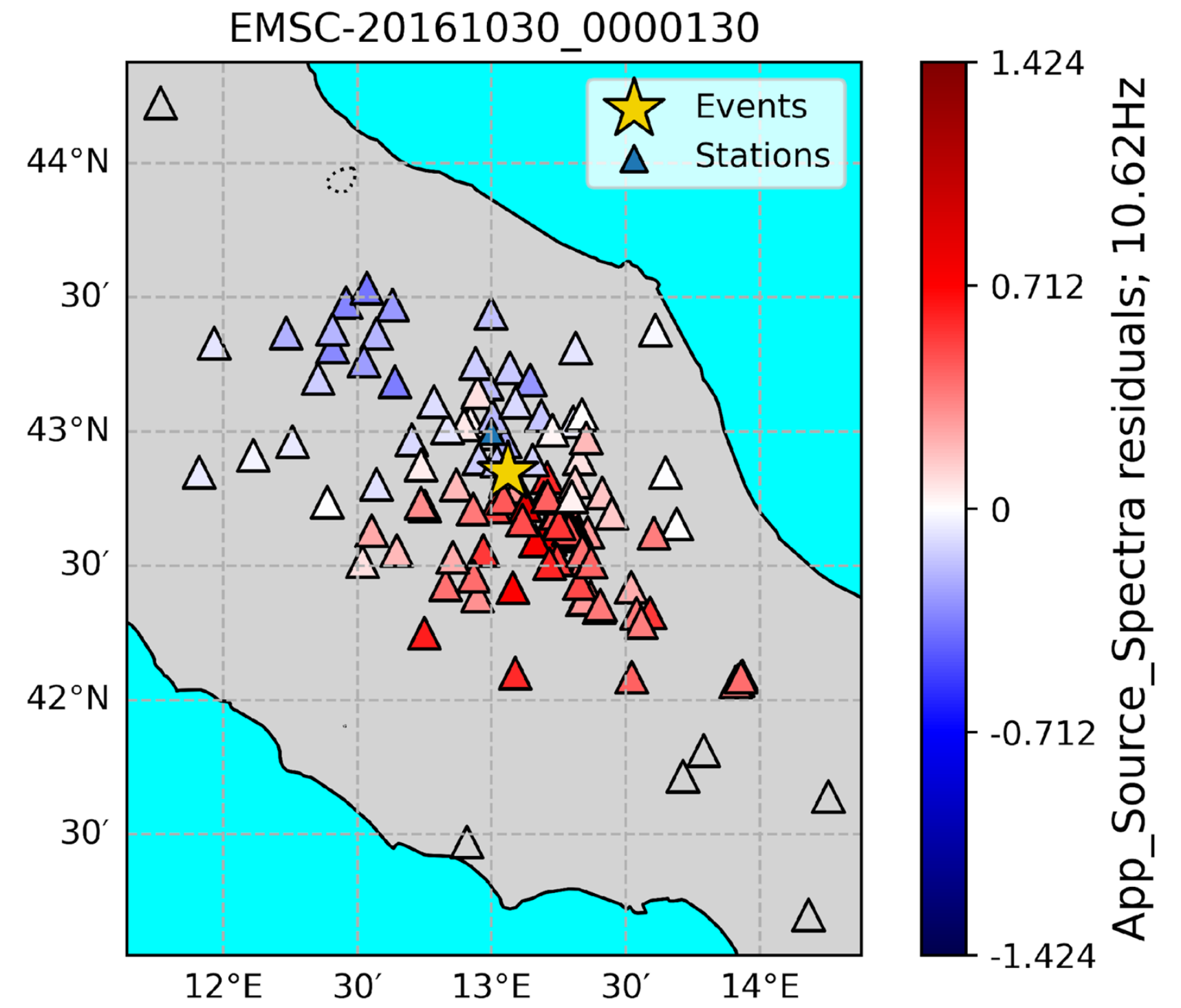
$$\text{Residuals} = \log (\text{AppSS}) - \log (\omega^2)$$

Note: red = positive, blue = negative

Spatial Variations of ω^2 model (Brune, 1970)



2.0 Hz



10.0 Hz

Residuals = $\log(\text{AppSS}) - \log(\omega^2)$

Note: red = positive, blue = negative

C_d^1 fitting on the f_c

- **Step-2:** fitting the AppSS residual amplitudes on f_c using the directivity coefficient C_d (Ben-Menahem, 1961) by the **apparent corner frequency (f_a)**

$$f_a = f_c C_d^1(\theta)$$

$$C_d = \sqrt{\frac{k^2}{[1 - \alpha \cos(\theta - \theta_0)]^2} + \frac{(1 - k)^2}{[1 + \alpha \cos(\theta - \theta_0)]^2}}$$

(Boatwright, 2007)

α = Mach number, or ratio between rupture and shear-wave velocities

θ and θ_0 = the azimuth of the station and the rupture direction

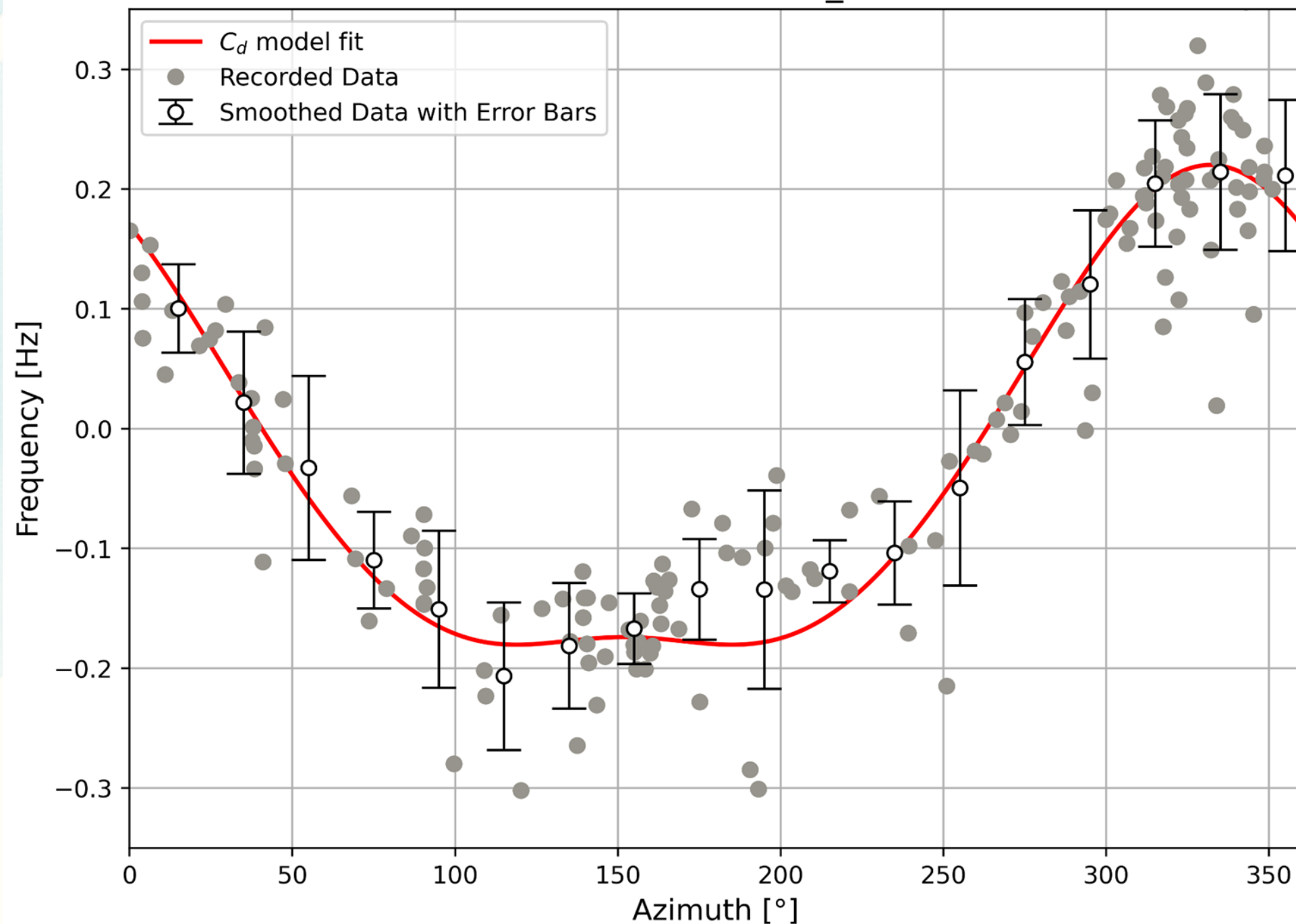
κ = the relative portion of the rupture length in θ_0 direction

- $n = 1$ for the linear Haskell model with constant slip and rise-time (Haskell, 1964)
- $n = 2$ for the k-squared model (Herrero & Bernard, 1994)

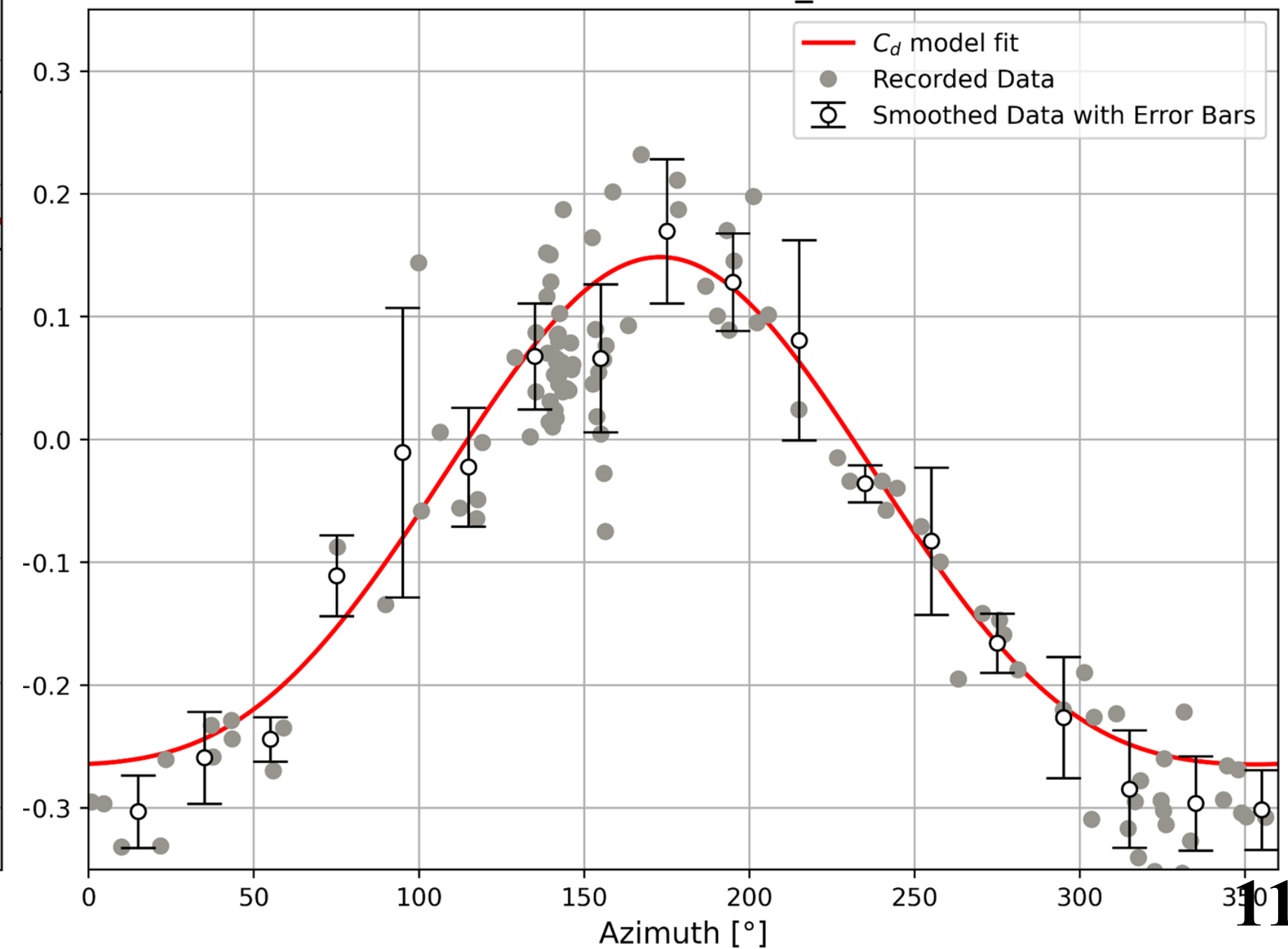
C_d^1 fitting on the f_c

Event Id	M_w	f_c	n	θ_0	α	κ	R^2
EMSC-20160903_0000009	4.2	1.11	1	332	0.5	0.79	0.98
EMSC-20161030_0000130	4.1	1.75		173	0.47	1	0.97

C_d functions for: EMSC-20160903_0000009; $M_w=4.35$



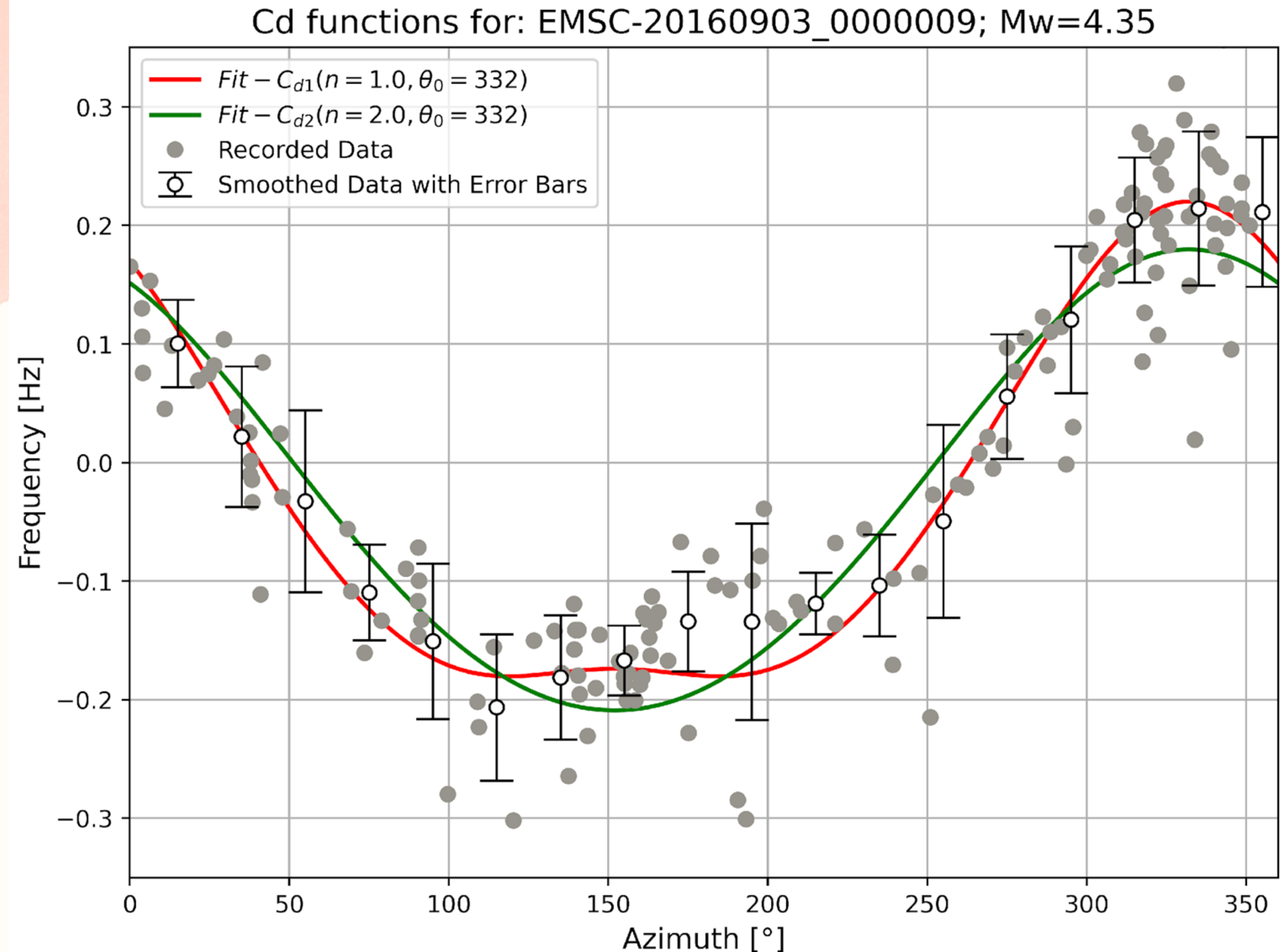
C_d functions for: EMSC-20161030_0000130; $M_w=4.10$



C_d^1 fitting on the f_c

	C_{d1}	C_{d2}
α	0.53	0.30
κ	0.80	0.94
n	1.0	2.0
θ_0	332	332
R^2	0.98	0.95

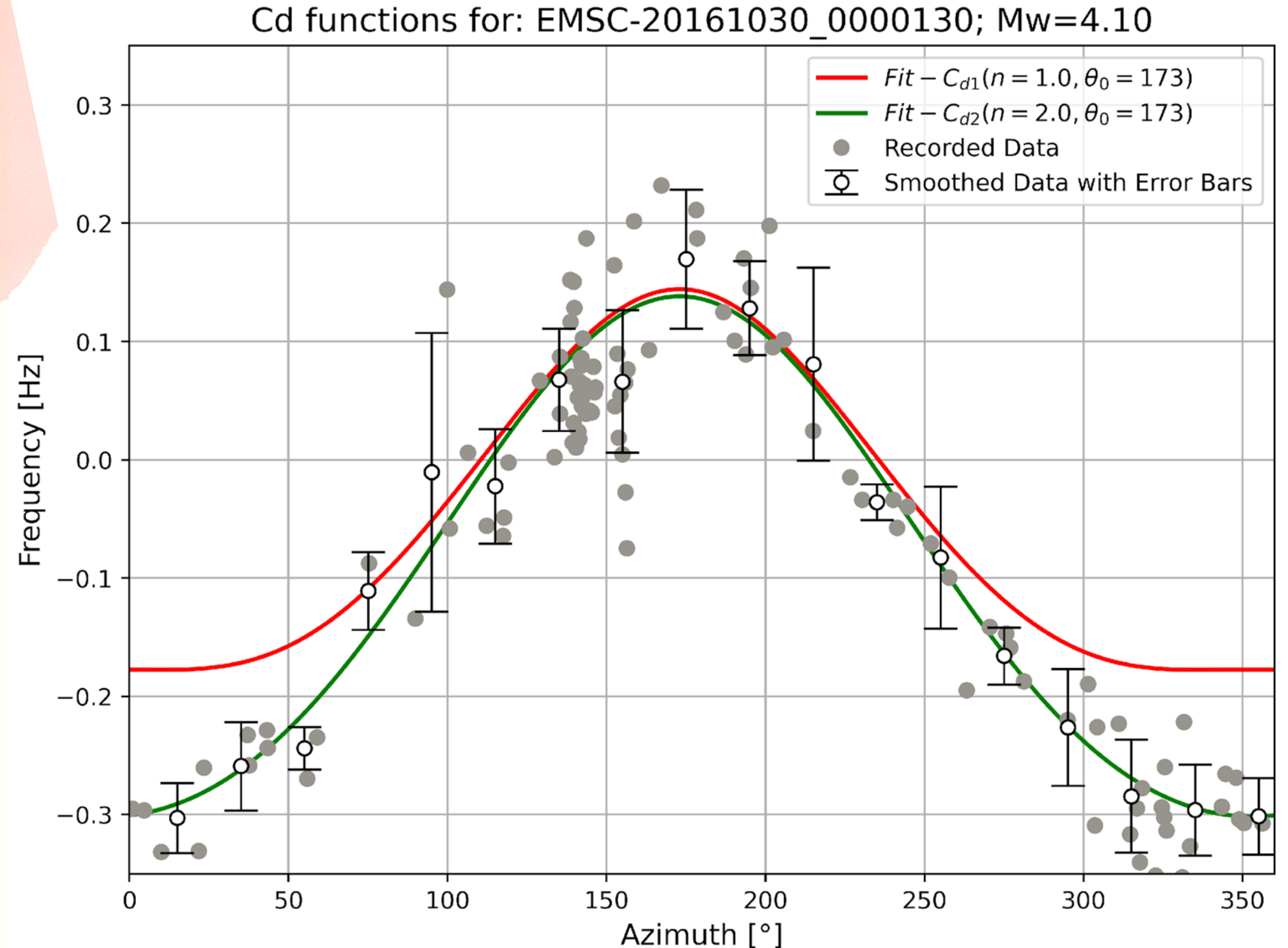
- There is a strong trade-off between n , κ and α .



C_d^1 fitting on the f_c

	C_{d1}	C_{d2}
α	0.44	0.35
κ	0.80	0.82
n	1.0	2.0
θ_0	173	173
R^2	0.83	0.96

- There is a strong trade-off between n and α .



C_d^n fitting on the Plateau

- **Step-3:** fitting the C_d^n function on the **plateau** ($f > f_c$) spectral ordinates, which implies that the amplitude of the plateau level (related to M_0) varies with azimuth due to directivity effects

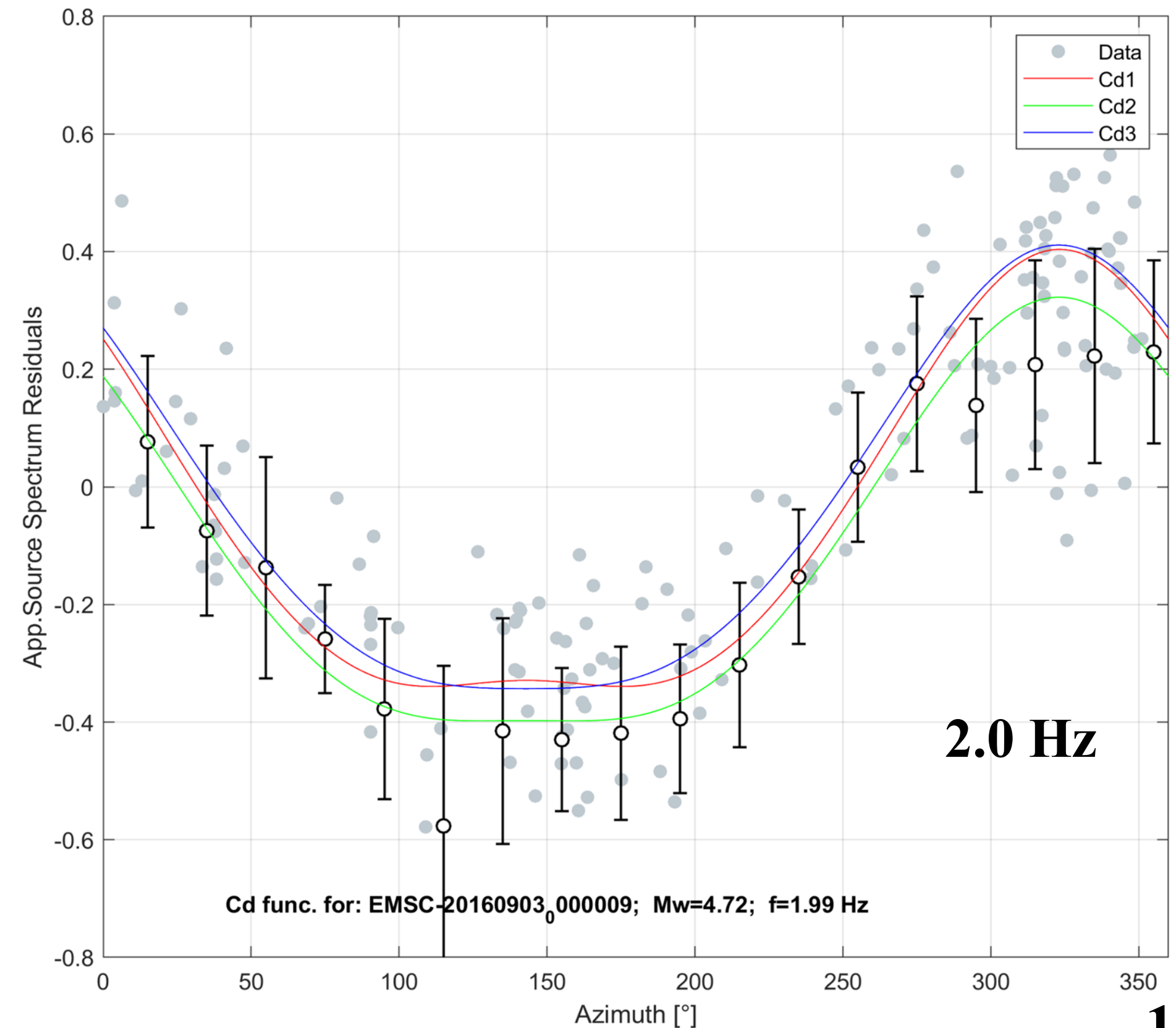
$$\omega_{\text{est}}^2 = \omega^2 \cdot C_d^n$$

** the goal is to find the **value(s)** of the directivity coefficient (C_{di}^n) that **best explain** these observed **variations** in the **plateau** level across different recording stations for a given earthquake*

C_d^n fitting on the Plateau

EMSC-

	C_{d1}	C_{d2}	C_{d3}
α	0.51	0.43	0.50
κ	0.80	0.79	0.85
n_{med}	1.71	1.92	1.62
n_{max}	2.02	2.29	1.90
θ_0	331	330	330
R^2	0.90	0.88	0.88

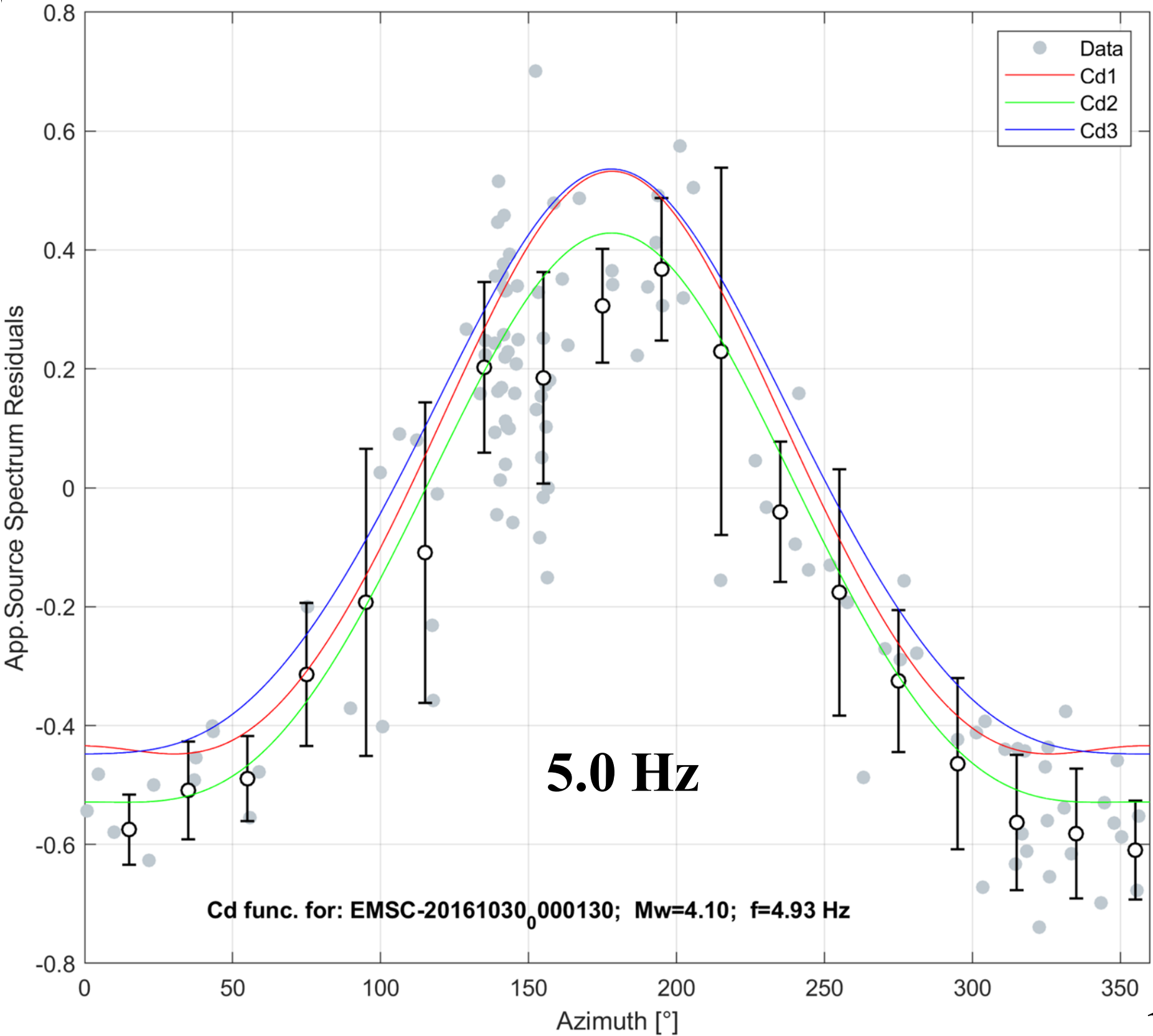


C_dⁿ fitting on the Plateau

EMSC-

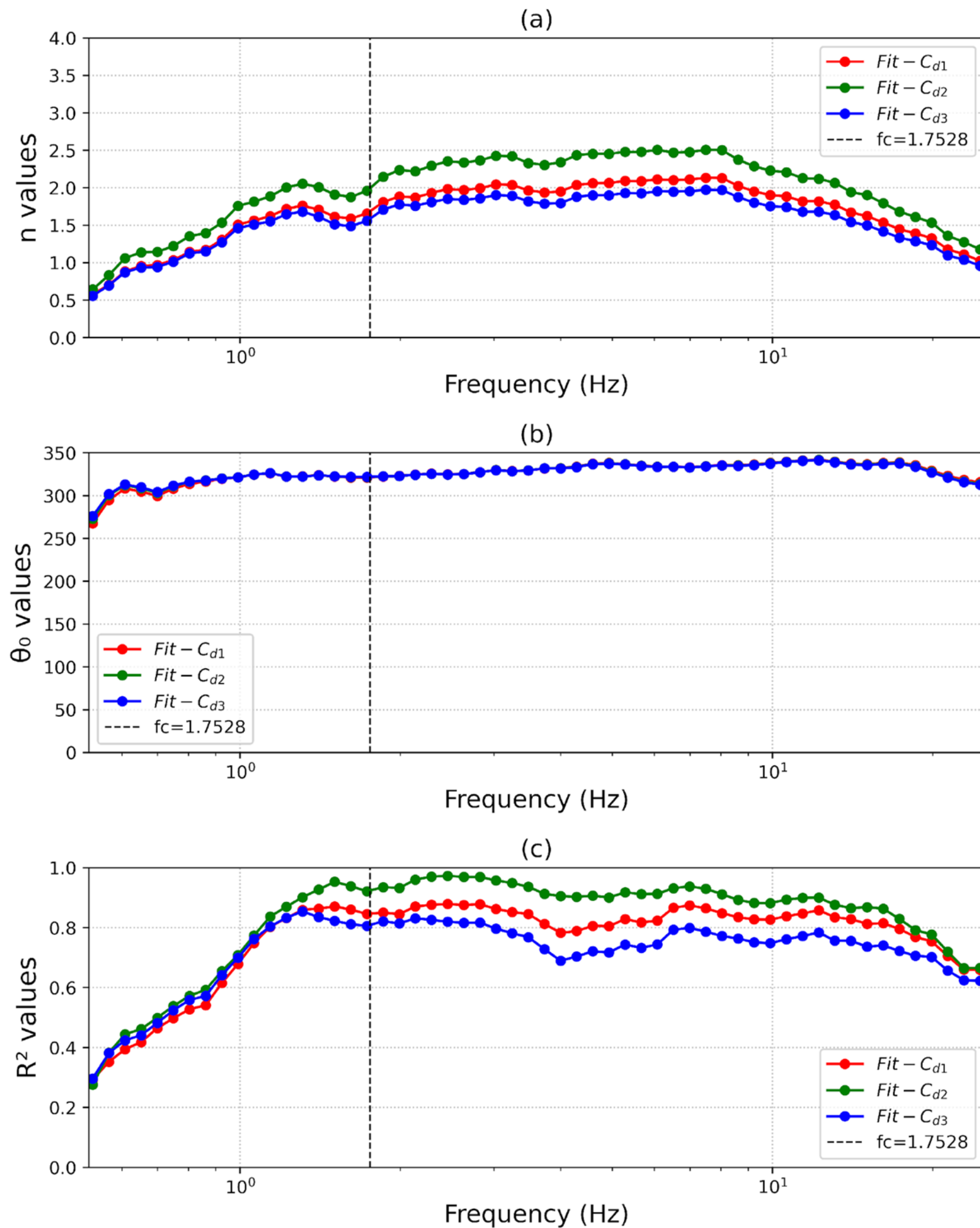
20161030_000130

	C _{d1}	C _{d2}	C _{d3}
α	0.51	0.43	0.50
κ	0.80	0.79	0.85
n_{med}	1.84	2.10	1.76
n_{max}	2.40	2.73	2.30
θ_0	172	172	172
R^2	0.91	0.90	0.91

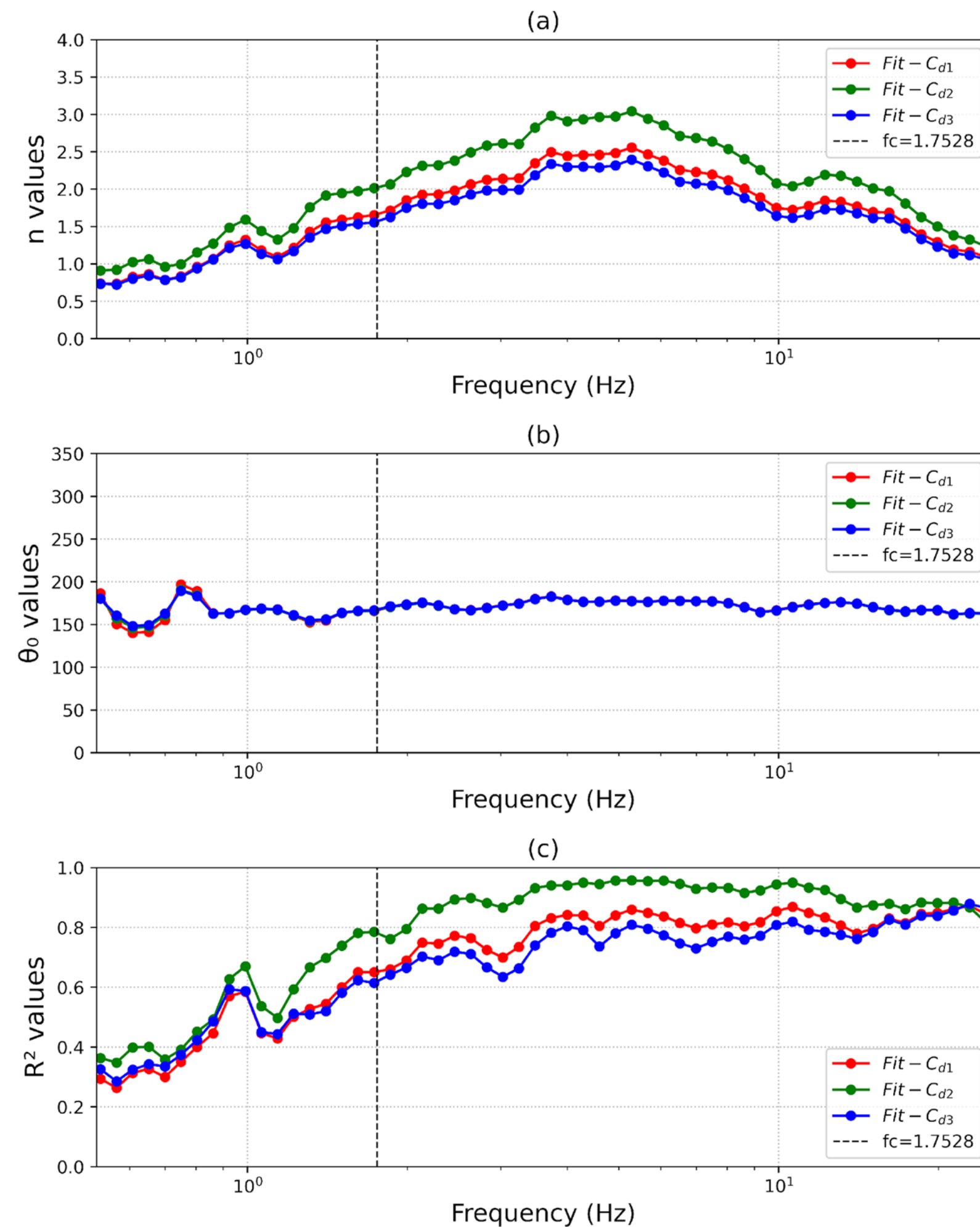


Parameters comparison

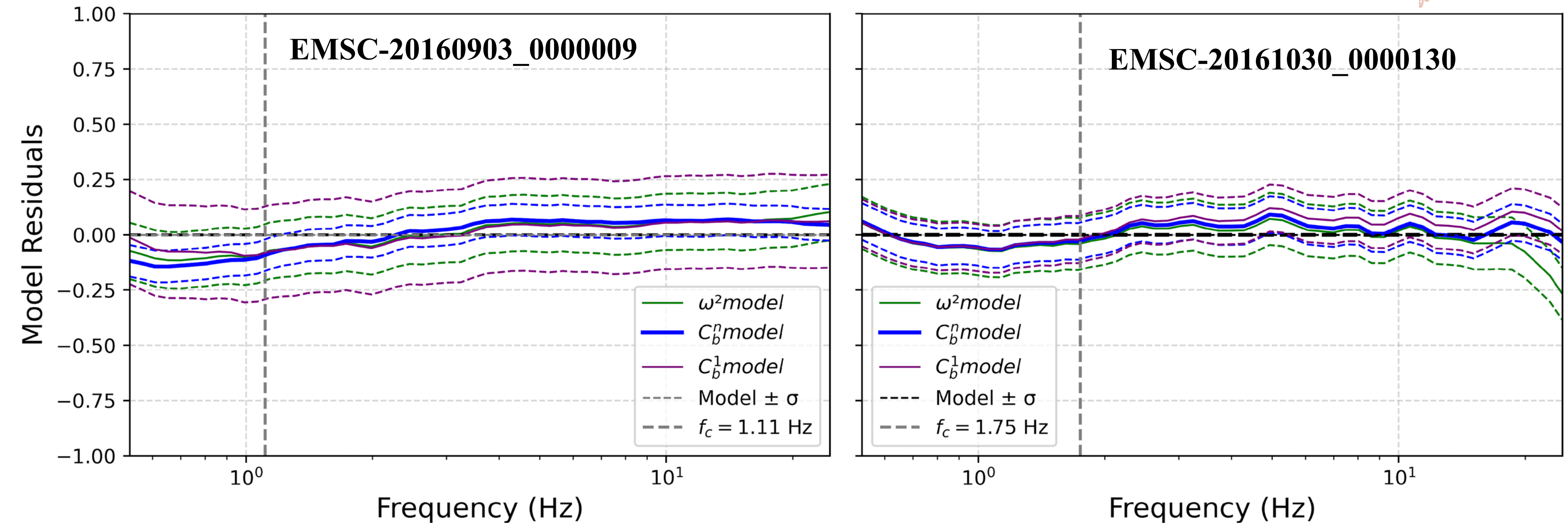
EMSC-20160903_0000009



EMSC-20161030_0000130



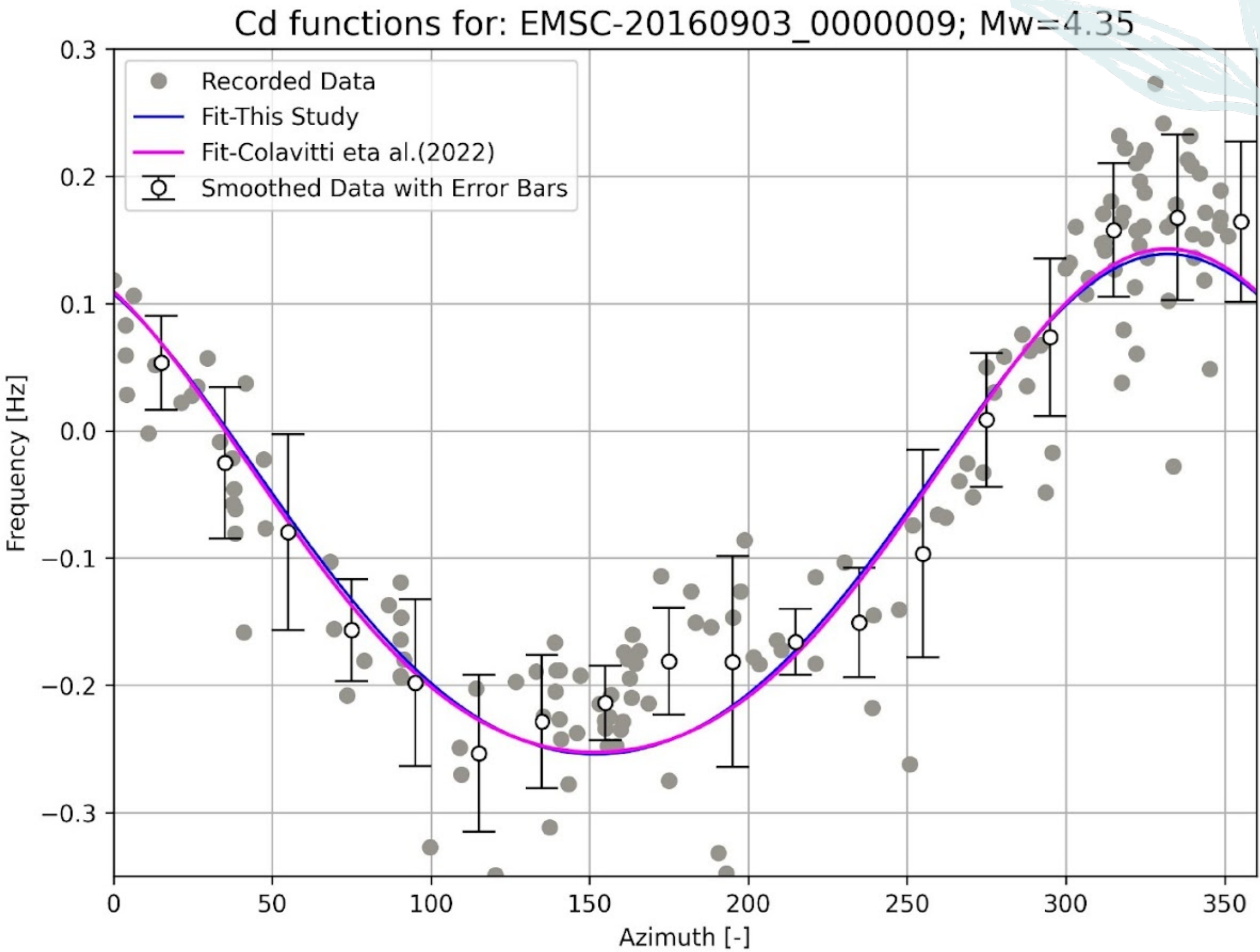
Models's Uncertainty



*AIC - Akaike Information Criterion (Akaike, 1974): C_b^n model (Plateau fit) ranked as the **best model***

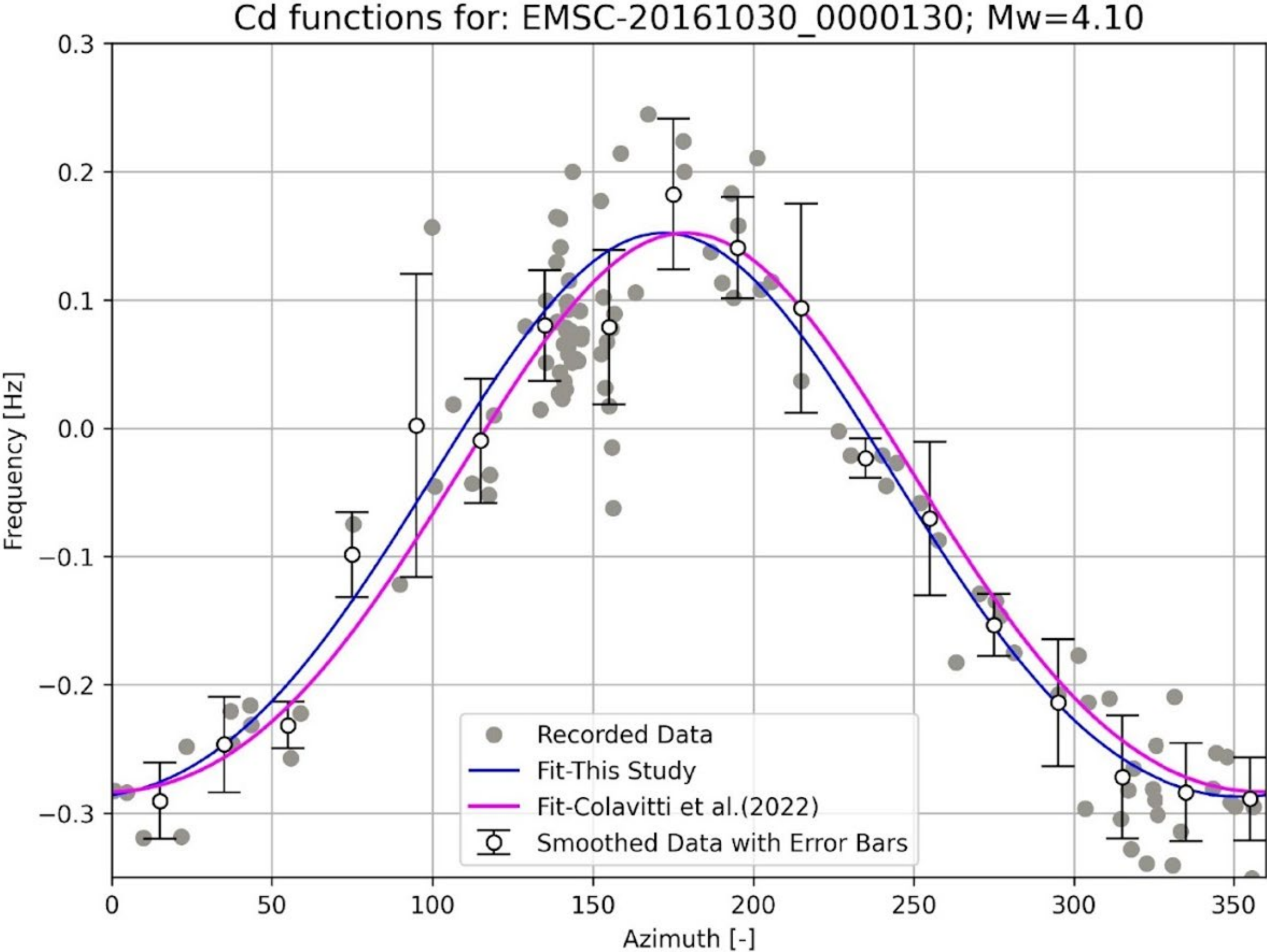
Colavitti et al.(2022) comparison

	This study	Colavitti et al. (2022)
α	0.51	0.50
κ	0.80	0.85
N	1.71	1.56
θ	331	331.92
R^2	0.98	-



Colavitti et al.(2022) comparison

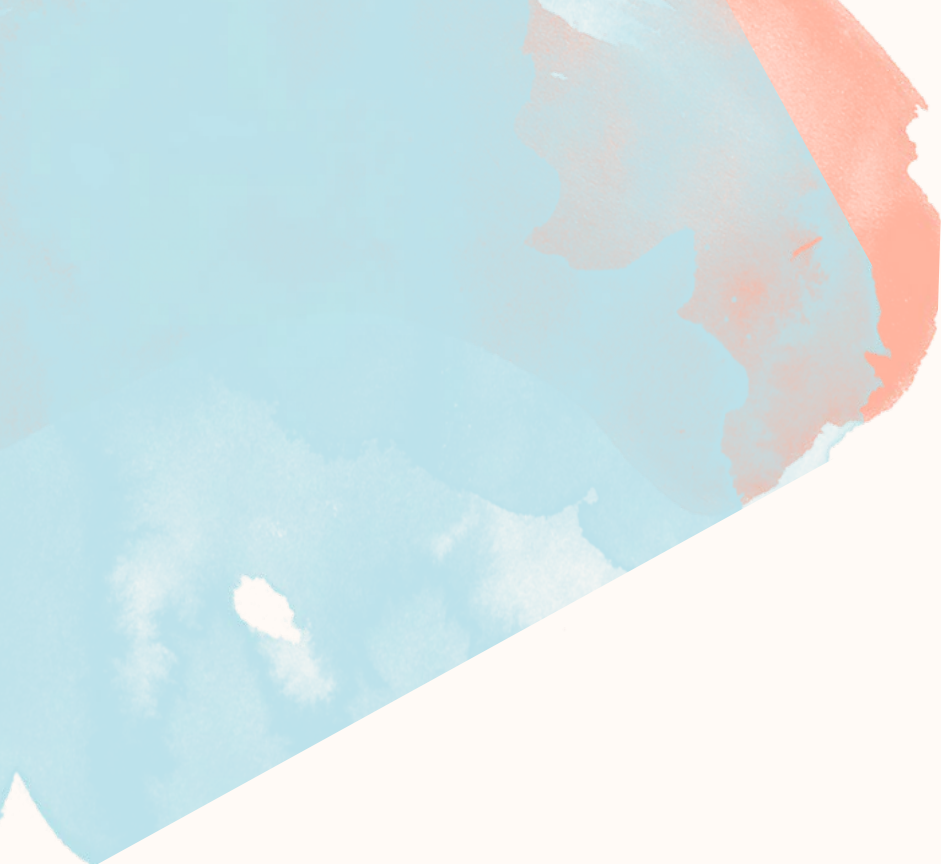
	This study	Colavitti et al. (2022)
α	0.51	0.50
κ	0.80	0.85
N	1.71	1.76
θ	173	178.87
R^2	0.98	-



Conclusions

- the rupture directivity effects may be identifiable even in small to moderate seismic events
- we adopt and test different models to capture empirical evidence of directivity in the Apparent Source Spectra (AppSS)
- residual analysis were conducted to evaluate the model performance and to check the accuracy of the adopted fitting
- comparison with prior studies confirms the reliability of the spectral estimates and modeling approach

Future work: Application of the approach/methodology with a larger dataset of small to moderate seismic events...



Thank you!